NIMBLE User Manual

NIMBLE Development Team

Version 0.13.1



https://r-nimble.org https://github.com/nimble-dev/nimble

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Part I Introduction

Chapter 1

Welcome to NIMBLE

NIMBLE is a system for building and sharing analysis methods for statistical models from R, especially for hierarchical models and computationally-intensive methods. While NIMBLE is embedded in R, it goes beyond R by supporting separate programming of models and algorithms along with compilation for fast execution.

As of version 0.13.2, NIMBLE has been around for a while and is reasonably stable, but we have a lot of plans to expand and improve it. The algorithm library provides MCMC with a lot of user control and ability to write new samplers easily. Other algorithms include particle filtering (sequential Monte Carlo) and Monte Carlo Expectation Maximization (MCEM).

But NIMBLE is about much more than providing an algorithm library. It provides a language for writing model-generic algorithms. We hope you will program in NIMBLE and make an R package providing your method. Of course, NIMBLE is open source, so we also hope you'll contribute to its development.

Please join the mailing lists (see R-nimble.org/more/issues-and-groups) and help improve NIMBLE by telling us what you want to do with it, what you like, and what could be better. We have a lot of ideas for how to improve it, but we want your help and ideas too. You can also follow and contribute to developer discussions on our GitHub repository.

If you use NIMBLE in your work, please cite us, as this helps justify past and future funding for the development of NIMBLE. For more information, please call citation('nimble') in R.

1.1 What does NIMBLE do?

NIMBLE makes it easier to program statistical algorithms that will run efficiently and work on many different models from R.

You can think of NIMBLE as comprising four pieces:

- 1. A system for writing statistical models flexibly, which is an extension of the BUGS language¹.
- 2. A library of algorithms such as MCMC.
- 3. A language, called NIMBLE, embedded within and similar in style to R, for writing algorithms that operate on models written in BUGS.

¹See Chapter 5 for information about NIMBLE's version of BUGS.

4. A compiler that generates C++ for your models and algorithms, compiles that C++, and lets you use it seamlessly from R without knowing anything about C++.

NIMBLE stands for Numerical Inference for statistical Models for Bayesian and Likelihood Estimation.

Although NIMBLE was motivated by algorithms for hierarchical statistical models, it's useful for other goals too. You could use it for simpler models. And since NIMBLE can automatically compile R-like functions into C++ that use the Eigen library for fast linear algebra, you can use it to program fast numerical functions without any model involved².

One of the beauties of R is that many of the high-level analysis functions are themselves written in R, so it is easy to see their code and modify them. The same is true for NIMBLE: the algorithms are themselves written in the NIMBLE language.

1.2 How to use this manual

We suggest everyone start with the Lightning Introduction in Chapter 2.

Then, if you want to jump into using NIMBLE's algorithms without learning about NIMBLE's programming system, go to Part II to learn how to build your model and Part III to learn how to apply NIMBLE's built-in algorithms to your model.

If you want to learn about NIMBLE programming (nimbleFunctions), go to Part IV. This teaches how to program user-defined function or distributions to use in BUGS code, compile your R code for faster operations, and write algorithms with NIMBLE. These algorithms could be specific algorithms for your particular model (such as a user-defined MCMC sampler for a parameter in your model) or general algorithms you can distribute to others. In fact the algorithms provided as part of NIMBLE and described in Part III are written as nimbleFunctions.

 $^{^2}$ The packages Rcpp and RcppEigen provide different ways of connecting C++, the Eigen library and R. In those packages you program directly in C++, while in NIMBLE you program in R in a nimbleFunction and the NIMBLE compiler turns it into C++.

Chapter 2

Lightning introduction

2.1 A brief example

Here we'll give a simple example of building a model and running some algorithms on the model, as well as creating our own user-specified algorithm. The goal is to give you a sense for what one can do in the system. Later sections will provide more detail.

We'll use the *pump* model example from BUGS¹. We could load the model from the standard BUGS example file formats (Section 6.1.2), but instead we'll show how to enter it directly in R.

In this 'lightning introduction' we will:

- 1. Create the model for the pump example.
- 2. Compile the model.
- 3. Create a basic MCMC configuration for the pump model.
- 4. Compile and run the MCMC
- 5. Customize the MCMC configuration and compile and run that.
- 6. Create, compile and run a Monte Carlo Expectation Maximization (MCEM) algorithm, which illustrates some of the flexibility NIMBLE provides to combine R and NIMBLE.
- 7. Write a short nimbleFunction to generate simulations from designated nodes of any model.

2.2 Creating a model

First we define the model code, its constants, data, and initial values for MCMC.

```
pumpCode <- nimbleCode({
  for (i in 1:N){
     theta[i] ~ dgamma(alpha,beta)
     lambda[i] <- theta[i]*t[i]
     x[i] ~ dpois(lambda[i])
}
alpha ~ dexp(1.0)
beta ~ dgamma(0.1,1.0)
})</pre>
```

¹The data set describes failure rates of some pumps.

```
pump <- nimbleModel(code = pumpCode, name = "pump", constants = pumpConsts,</pre>
                   data = pumpData, inits = pumpInits)
pump$getNodeNames()
   [1] "alpha"
                             "beta"
                                                  "lifted_d1_over_beta"
##
   [4] "theta[1]"
                             "theta[2]"
                                                  "theta[3]"
##
  [7] "theta[4]"
                             "theta[5]"
                                                  "theta[6]"
## [10] "theta[7]"
                             "theta[8]"
                                                  "theta[9]"
## [13] "theta[10]"
                             "lambda[1]"
                                                  "lambda[2]"
## [16] "lambda[3]"
                             "lambda[4]"
                                                  "lambda[5]"
## [19] "lambda[6]"
                             "lambda[7]"
                                                  "lambda[8]"
## [22] "lambda[9]"
                             "lambda[10]"
                                                  "x[1]"
## [25] "x[2]"
                             "x[3]"
                                                  "x[4]"
## [28] "x[5]"
                             "x[6]"
                                                  "x[7]"
## [31] "x[8]"
                             "x[9]"
                                                  "x[10]"
pump$x
## [1] 5 1 5 14 3 19 1 1 4 22
pump$logProb_x
   [1]
        -2.998011 -1.118924 -1.882686 -2.319466 -4.254550 -20.739651
   [7]
        -2.358795 -2.358795 -9.630645 -48.447798
pump$alpha
## [1] 1
pump$theta
  pump$lambda
```

```
## [1] 9.430 1.570 6.290 12.600 0.524 3.140 0.105 0.105 0.210 1.050
```

Notice that in the list of nodes, NIMBLE has introduced a new node, lifted_d1_over_beta. We

call this a 'lifted' node. Like R, NIMBLE allows alternative parameterizations, such as the scale or rate parameterization of the gamma distribution. Choice of parameterization can generate a lifted node, as can using a link function or a distribution argument that is an expression. It's helpful to know why they exist, but you shouldn't need to worry about them.

Thanks to the plotting capabilities of the **igraph** package that NIMBLE uses to represent the directed acyclic graph, we can plot the model (Figure 2.1).

pump\$plotGraph()

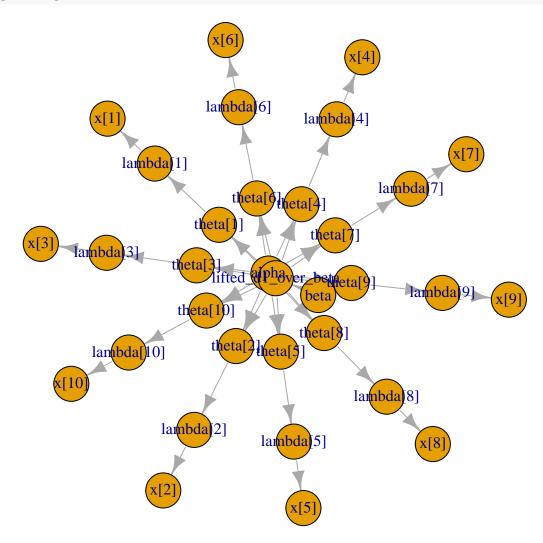


Figure 2.1: Directed Acyclic Graph plot of the pump model, thanks to the igraph package

You are in control of the model. By default, nimbleModel does its best to initialize a model, but let's say you want to re-initialize theta. To simulate from the prior for theta (overwriting the initial values previously in the model) we first need to be sure the parent nodes of all theta[i] nodes are fully initialized, including any non-stochastic nodes such as lifted nodes. We then use the simulate function to simulate from the distribution for theta. Finally we use the calculate function to calculate the dependencies of theta, namely lambda and the log probabilities of x to ensure all parts of the model are up to date. First we show how to use the model's getDependencies method to query information about its graph.

##

```
# Show all dependencies of alpha and beta terminating in stochastic nodes
pump$getDependencies(c("alpha", "beta"))
   [1] "alpha"
                             "beta"
                                                   "lifted_d1_over_beta"
  [4] "theta[1]"
                             "theta[2]"
                                                   "theta[3]"
## [7] "theta[4]"
                             "theta[5]"
                                                   "theta[6]"
## [10] "theta[7]"
                             "theta[8]"
                                                   "theta[9]"
## [13] "theta[10]"
# Now show only the deterministic dependencies
pump$getDependencies(c("alpha", "beta"), determOnly = TRUE)
## [1] "lifted_d1_over_beta"
# Check that the lifted node was initialized.
pump[["lifted_d1_over_beta"]] # It was.
## [1] 1
# Now let's simulate new theta values
set.seed(1) # This makes the simulations here reproducible
pump$simulate("theta")
pump$theta
           # the new theta values
   [1] 0.15514136 1.88240160 1.80451250 0.83617765 1.22254365 1.15835525
   [7] 0.99001994 0.30737332 0.09461909 0.15720154
# lambda and logProb_x haven't been re-calculated yet
pump$lambda # these are the same values as above
## [1] 9.430 1.570 6.290 12.600 0.524 3.140 0.105 0.105 0.210 1.050
pump$logProb_x
   [1] -2.998011 -1.118924 -1.882686 -2.319466 -4.254550 -20.739651
   [7] -2.358795 -2.358795 -9.630645 -48.447798
pump$getLogProb("x") # The sum of logProb_x
## [1] -96.10932
pump$calculate(pump$getDependencies(c("theta")))
## [1] -262.204
pump$lambda # Now they have.
   [1] 14.6298299 29.5537051 113.5038360 105.3583839
                                                         6.4061287 36.3723548
##
   [7]
         1.0395209
                     0.3227420
                                 0.1987001
                                             1.6506161
pump$logProb_x
  [1] -6.002009 -26.167496 -94.632145 -65.346457 -2.626123 -7.429868
##
```

[7] -1.000761 -1.453644 -9.840589 -39.096527

Notice that the first getDependencies call returned dependencies from alpha and beta down to the next stochastic nodes in the model. The second call requested only deterministic dependencies. The call to pump\$simulate("theta") expands "theta" to include all nodes in theta. After simulating into theta, we can see that lambda and the log probabilities of x still reflect the old values of theta, so we calculate them and then see that they have been updated.

2.3 Compiling the model

Next we compile the model, which means generating C++ code, compiling that code, and loading it back into R with an object that can be used just like the uncompiled model. The values in the compiled model will be initialized from those of the original model in R, but the original and compiled models are distinct objects so any subsequent changes in one will not be reflected in the other.

```
Cpump <- compileNimble(pump)
Cpump$theta
## [1] 0.15514136 1.88240160 1.80451250 0.83617765 1.22254365 1.15835525</pre>
```

Note that the compiled model is used when running any NIMBLE algorithms via C++, so the model needs to be compiled before (or at the same time as) any compilation of algorithms, such as the compilation of the MCMC done in the next section.

2.4 One-line invocation of MCMC

[7] 0.99001994 0.30737332 0.09461909 0.15720154

The most direct approach to invoking NIMBLE's MCMC engine is using the nimbleMCMC function. This function would generally take the code, data, constants, and initial values as input, but it can also accept the (compiled or uncompiled) model object as an argument. It provides a variety of options for executing and controlling multiple chains of NIMBLE's default MCMC algorithm, and returning posterior samples, posterior summary statistics, and/or WAIC values.

For example, to execute two MCMC chains of 10,000 samples each, and return samples, summary statistics, and WAIC values:

[Warning] There are individual pWAIC values that are greater than 0.4. This may indicate names(mcmc.out)

```
## [1] "samples" "summary" "WAIC"

mcmc.out$summary

## $chain1
## Mean Median St.Dev. 95%CI_low 95%CI_upp
```

```
## alpha
            0.69804352 0.65835063 0.27037676 0.287898244 1.3140461
## beta
            0.92862598 0.82156847 0.54969128 0.183699137 2.2872696
## theta[1] 0.06019274 0.05676327 0.02544956 0.021069950 0.1199230
## theta[2] 0.10157737 0.08203988 0.07905076 0.008066869 0.3034085
## theta[3] 0.08874755 0.08396502 0.03760562 0.031186960 0.1769982
## theta[4] 0.11567784 0.11301465 0.03012598 0.064170937 0.1824525
## theta[5] 0.60382223 0.54935089 0.31219612 0.159731108 1.3640771
## theta[6] 0.61204831 0.60085518 0.13803302 0.372712375 0.9135269
## theta[7] 0.90263434 0.70803389 0.73960182 0.074122175 2.7598261
## theta[8] 0.89021051 0.70774794 0.72668155 0.072571029 2.8189252
## theta[9] 1.57678136 1.44390008 0.76825189 0.455195149 3.4297368
## theta[10] 1.98954127 1.96171250 0.42409802 1.241383787 2.9012192
##
## $chain2
##
                  Mean
                           Median
                                     St.Dev.
                                               95%CI_low 95%CI_upp
## alpha
            0.69101961 0.65803654 0.26548378 0.277195564 1.2858148
## beta
            0.91627273 0.81434426 0.53750825 0.185772263 2.2702428
## theta[1] 0.05937364 0.05611283 0.02461866 0.020956151 0.1161870
## theta[2] 0.10017726 0.08116259 0.07855024 0.008266343 0.3010355
## theta[3] 0.08908126 0.08390782 0.03704170 0.031330829 0.1736876
## theta[4] 0.11592652 0.11356920 0.03064645 0.063595333 0.1829574
## theta[5] 0.59755632 0.54329373 0.31871551 0.149286703 1.3748728
## theta[6] 0.61080189 0.59946693 0.13804343 0.371373877 0.9097319
## theta[7] 0.89902759 0.70901502 0.72930369 0.076243503 2.7441445
## theta[8] 0.89954594 0.70727079 0.73345905 0.071250926 2.8054633
## theta[9] 1.57530029 1.45005738 0.75242164 0.469959364 3.3502795
## theta[10] 1.98911473 1.96227061 0.42298189 1.246910723 2.9102326
##
## $all.chains
##
                           Median
                                     St.Dev. 95%CI_low 95%CI_upp
                  Mean
## alpha
            0.69453156 0.65803654 0.26795776 0.28329854 1.2999319
            0.92244935 0.81828160 0.54365539 0.18549077 2.2785444
## beta
## theta[1] 0.05978319 0.05646474 0.02504028 0.02102807 0.1183433
## theta[2] 0.10087731 0.08162361 0.07880204 0.00811108 0.3017967
## theta[3] 0.08891440 0.08394667 0.03732417 0.03123228 0.1749967
## theta[4] 0.11580218 0.11326039 0.03038683 0.06385253 0.1827382
## theta[5] 0.60068928 0.54668011 0.31548032 0.15363752 1.3686801
## theta[6] 0.61142510 0.60015416 0.13803618 0.37203765 0.9122467
## theta[7] 0.90083096 0.70852800 0.73445465 0.07550465 2.7534885
## theta[8] 0.89487822 0.70761105 0.73007484 0.07211191 2.8067373
## theta[9] 1.57604083 1.44719278 0.76035931 0.46374515 3.3866706
## theta[10] 1.98932800 1.96195345 0.42352979 1.24334249 2.9068229
mcmc.out$WAIC
```

```
## nimbleList object of type waicList
## Field "WAIC":
## [1] 48.69896
```

```
## Field "lppd":

## [1] -19.99653

## Field "pWAIC":

## [1] 4.352945
```

See Section 7.1 or help(nimbleMCMC) for more details about using nimbleMCMC.

pumpConf <- configureMCMC(pump, print = TRUE)</pre>

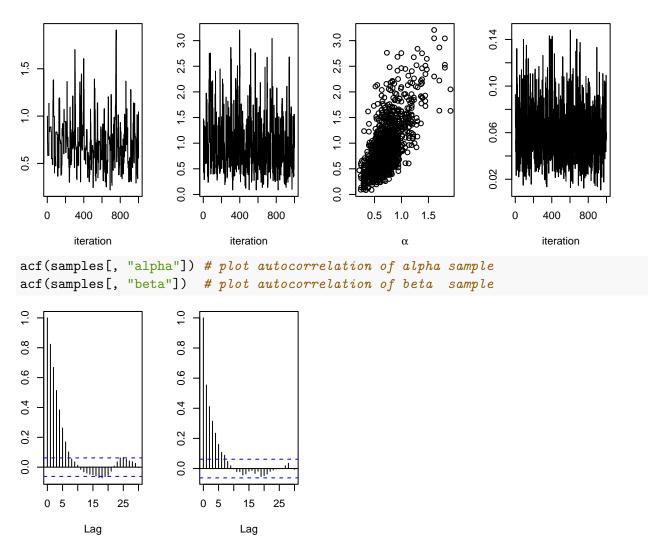
Note that the WAIC value varies depending on what quantities are treated as parameters. See Section 7.7 or help(waic) for more details.

2.5 Creating, compiling and running a basic MCMC configuration

At this point we have initial values for all of the nodes in the model, and we have both the original and compiled versions of the model. As a first algorithm to try on our model, let's use NIMBLE's default MCMC. Note that conjugate relationships are detected for all nodes except for alpha, on which the default sampler is a random walk Metropolis sampler.

```
## ==== Monitors =====
## thin = 1: alpha, beta
## ==== Samplers =====
## RW sampler (1)
## - alpha
## conjugate sampler (11)
## - beta
```

```
## - theta[] (10 elements)
pumpConf$addMonitors(c("alpha", "beta", "theta"))
```



Notice the posterior correlation between alpha and beta. A measure of the mixing for each is the autocorrelation for each parameter, shown by the acf plots.

2.6 Customizing the MCMC

Let's add an adaptive block sampler on alpha and beta jointly and see if that improves the mixing.

NULL

```
samplesNew <- as.matrix(CpumpNewMCMC$mvSamples)</pre>
par(mfrow = c(1, 4), mai = c(.6, .4, .1, .2))
plot(samplesNew[ , "alpha"], type = "l", xlab = "iteration",
     ylab = expression(alpha))
plot(samplesNew[ , "beta"], type = "l", xlab = "iteration",
     ylab = expression(beta))
plot(samplesNew[ , "alpha"], samplesNew[ , "beta"], xlab = expression(alpha),
     ylab = expression(beta))
plot(samplesNew[ , "theta[1]"], type = "l", xlab = "iteration",
     ylab = expression(theta[1]))
                                                                    0.15
                                             က
1.5
0.
                                                                    0.05
                                                  0.5
                                                         1.5
        400
             800
                          0
                               400
                                    800
        iteration
                              iteration
                                                                            iteration
                                                       α
acf(samplesNew[, "alpha"]) # plot autocorrelation of alpha sample
acf(samplesNew[, "beta"]) # plot autocorrelation of beta sample
0.8
                      0.8
9.0
                      9.0
0.4
                      9.4
   0 5
          15
              25
                          0 5
                                    25
                                15
         Lag
                                Lag
```

We can see that the block sampler has decreased the autocorrelation for both alpha and beta. Of course these are just short runs, and what we are really interested in is the effective sample size of the MCMC per computation time, but that's not the point of this example.

Once you learn the MCMC system, you can write your own samplers and include them. The entire system is written in nimbleFunctions.

2.7 Running MCEM

NIMBLE is a system for working with algorithms, not just an MCMC engine. So let's try maximizing the marginal likelihood for alpha and beta using Monte Carlo Expectation Maximization².

```
pump2 <- pump$newModel()</pre>
box = list( list(c("alpha", "beta"), c(0, Inf)))
pumpMCEM <- buildMCEM(model = pump2, latentNodes = "theta[1:10]",</pre>
                       boxConstraints = box)
pumpMLE <- pumpMCEM$run()</pre>
## Iteration Number: 1.
## Current number of MCMC iterations: 1000.
## Parameter Estimates:
##
       alpha
                  beta
## 0.8160625 1.1230921
## Convergence Criterion: 1.001.
## Iteration Number: 2.
## Current number of MCMC iterations: 1000.
## Parameter Estimates:
##
       alpha
                  beta
## 0.8045037 1.1993128
## Convergence Criterion: 0.0223464.
## Iteration Number: 3.
## Current number of MCMC iterations: 1250.
## Parameter Estimates:
       alpha
                  beta
## 0.8203178 1.2497067
## Convergence Criterion: 0.004913688.
## Iteration Number: 4.
## Current number of MCMC iterations: 3032.
## Parameter Estimates:
##
       alpha
                  beta
## 0.8226618 1.2602452
## Convergence Criterion: 0.0004201045.
pumpMLE
```

```
## alpha beta
## 0.8226618 1.2602452
```

Both estimates are within 0.01 of the values reported by George et al. (1993)³. Some discrepancy is to be expected since it is a Monte Carlo algorithm.

 $^{^{2}}$ Note that for this model, one could analytically integrate over theta and then numerically maximize the resulting marginal likelihood.

³Table 2 of the paper accidentally swapped the two estimates.

2.8 Creating your own functions

Now let's see an example of writing our own algorithm and using it on the model. We'll do something simple: simulating multiple values for a designated set of nodes and calculating every part of the model that depends on them. More details on programming in NIMBLE are in Part IV.

Here is our *nimbleFunction*:

```
simNodesMany <- nimbleFunction(</pre>
    setup = function(model, nodes) {
        mv <- modelValues(model)</pre>
        deps <- model$getDependencies(nodes)</pre>
        allNodes <- model$getNodeNames()</pre>
    },
    run = function(n = integer()) {
        resize(mv, n)
        for(i in 1:n) {
             model$simulate(nodes)
             model$calculate(deps)
             copy(from = model, nodes = allNodes,
                  to = mv, rowTo = i, logProb = TRUE)
         }
    })
simNodesTheta1to5 <- simNodesMany(pump, "theta[1:5]")</pre>
simNodesTheta6to10 <- simNodesMany(pump, "theta[6:10]")</pre>
```

Here are a few things to notice about the nimbleFunction.

- 1. The setup function is written in R. It creates relevant information specific to our model for use in the run-time code.
- 2. The setup code creates a *modelValues* object to hold multiple sets of values for variables in the model provided.
- 3. The run function is written in NIMBLE. It carries out the calculations using the information determined once for each set of model and nodes arguments by the setup code. The run-time code is what will be compiled.
- 4. The run code requires type information about the argument n. In this case it is a scalar integer.
- 5. The for-loop looks just like R, but only sequential integer iteration is allowed.
- 6. The functions calculate and simulate, which were introduced above in R, can be used in NIMBLE
- 7. The special function copy is used here to record values from the model into the modelValues object.
- 8. Multiple instances, or 'specializations', can be made by calling simNodesMany with different arguments. Above, simNodesTheta1to5 has been made by calling simNodesMany with the pump

model and nodes "theta[1:5]" as inputs to the setup function, while simNodesTheta6to10 differs by providing "theta[6:10]" as an argument. The returned objects are objects of a uniquely generated R reference class with fields (member data) for the results of the setup code and a run method (member function).

By the way, simNodesMany is very similar to a standard nimbleFunction provided with NIMBLE, simNodesMV.

Now let's execute this nimbleFunction in R, before compiling it.

```
set.seed(1) # make the calculation repeatable
pump$alpha <- pumpMLE[1]</pre>
pump$beta <- pumpMLE[2]</pre>
# make sure to update deterministic dependencies of the altered nodes
pump$calculate(pump$getDependencies(c("alpha","beta"), determOnly = TRUE))
## [1] 0
saveTheta <- pump$theta
simNodesTheta1to5$run(10)
simNodesTheta1to5$mv[["theta"]][1:2]
## [[1]]
    [1] 0.21829875 1.93210970 0.62296551 0.34197267 3.45729603 1.15835525
    [7] 0.99001994 0.30737332 0.09461909 0.15720154
##
##
## [[2]]
   [1] 0.82759982 0.08784057 0.34414959 0.29521943 0.14183505 1.15835525
##
   [7] 0.99001994 0.30737332 0.09461909 0.15720154
simNodesTheta1to5$mv[["logProb_x"]][1:2]
## [[1]]
   [1] -10.250111 -26.921849 -25.630612 -15.594173 -11.217566 -7.429868
   [7] -1.000761 -1.453644 -9.840589 -39.096527
##
##
## [[2]]
   [1] -61.043876 -1.057668 -11.060164 -11.761432
                                                     -3.425282 -7.429868
##
        -1.000761
                   -1.453644 -9.840589 -39.096527
```

In this code we have initialized the values of alpha and beta to their MLE and then recorded the theta values to use below. Then we have requested 10 simulations from simNodesTheta1to5. Shown are the first two simulation results for theta and the log probabilities of x. Notice that theta[6:10] and the corresponding log probabilities for x[6:10] are unchanged because the nodes being simulated are only theta[1:5]. In R, this function runs slowly.

Finally, let's compile the function and run that version.

```
Cpump$calculate(Cpump$getDependencies(c("alpha","beta"), determOnly = TRUE))
## [1] 0
Cpump$theta <- saveTheta
set.seed(1)
CsimNodesTheta1to5$run(10)
## NULL
CsimNodesTheta1to5$mv[["theta"]][1:2]
## [[1]]
## [1] 0.21829875 1.93210970 0.62296551 0.34197267 3.45729603 1.15835525
## [7] 0.99001994 0.30737332 0.09461909 0.15720154
##
## [[2]]
## [1] 0.82759982 0.08784057 0.34414959 0.29521943 0.14183505 1.15835525
## [7] 0.99001994 0.30737332 0.09461909 0.15720154
CsimNodesTheta1to5$mv[["logProb_x"]][1:2]
## [[1]]
## [1] -10.250111 -26.921849 -25.630612 -15.594173 -11.217566 -2.782156
## [7] -1.042151 -1.004362 -1.894675 -3.081102
##
## [[2]]
## [1] -61.043876 -1.057668 -11.060164 -11.761432 -3.425282 -2.782156
## [7] -1.042151 -1.004362 -1.894675 -3.081102
```

Given the same initial values and the same random number generator seed, we got identical results for theta[1:5] and their dependencies, but it happened much faster.

Chapter 3

More introduction

Now that we have shown a brief example, we will introduce more about the concepts and design of NIMBLE.

One of the most important concepts behind NIMBLE is to allow a combination of high-level processing in R and low-level processing in C++. For example, when we write a Metropolis-Hastings MCMC sampler in the NIMBLE language, the inspection of the model structure related to one node is done in R, and the actual sampler calculations are done in C++. This separation between setup and run steps will become clearer as we go.

3.1 NIMBLE adopts and extends the BUGS language for specifying models

We adopted the BUGS language, and we have extended it to make it more flexible. The BUGS language became widely used in WinBUGS, then in OpenBUGS and JAGS. These systems all provide automatically-generated MCMC algorithms, but we have adopted only the language for describing models, not their systems for generating MCMCs.

NIMBLE extends BUGS by:

- 1. allowing you to write new functions and distributions and use them in BUGS models;
- 2. allowing you to define multiple models in the same code using conditionals evaluated when the BUGS code is processed;
- 3. supporting a variety of more flexible syntax such as R-like named parameters and more general algebraic expressions.

By supporting new functions and distributions, NIMBLE makes BUGS an extensible language, which is a major departure from previous packages that implement BUGS.

We adopted BUGS because it has been so successful, with over 30,000 users by the time they stopped counting (Lunn et al., 2009). Many papers and books provide BUGS code as a way to document their statistical models. We describe NIMBLE's version of BUGS later. The web sites for WinBUGS, OpenBUGS and JAGS provide other useful documntation on writing models in BUGS. For the most part, if you have BUGS code, you can try NIMBLE.

NIMBLE does several things with BUGS code:

- 1. NIMBLE creates a *model definition* object that knows everything about the variables and their relationships written in the BUGS code. Usually you'll ignore the *model definition* and let NIMBLE's default options take you directly to the next step.
- 2. NIMBLE creates a model object¹. This can be used to manipulate variables and operate the model from R. Operating the model includes calculating, simulating, or querying the log probability value of model nodes. These basic capabilities, along with the tools to query model structure, allow one to write programs that use the model and adapt to its structure.
- 3. When you're ready, NIMBLE can generate customized C++ code representing the model, compile the C++, load it back into R, and provide a new model object that uses the compiled model internally. We use the word 'compile' to refer to all of these steps together.

As an example of how radical a departure NIMBLE is from previous BUGS implementations, consider a situation where you want to simulate new data from a model written in BUGS code. Since NIMBLE creates model objects that you can control from R, simulating new data is trivial. With previous BUGS-based packages, this isn't possible.

More information about specifying and manipulating models is in Chapters 6 and 13.

3.2 nimbleFunctions for writing algorithms

NIMBLE provides *nimbleFunctions* for writing functions that can (but don't have to) use BUGS models. The main ways that nimbleFunctions can use BUGS models are:

- 1. inspecting the structure of a model, such as determining the dependencies between variables, in order to do the right calculations with each model;
- 2. accessing values of the model's variables;
- 3. controlling execution of the model's probability calculations or corresponding simulations;
- 4. managing model Values data structures for multiple sets of model values and probabilities.

In fact, the calculations of the model are themselves constructed as nimbleFunctions, as are the algorithms provided in NIMBLE's algorithm library².

Programming with nimbleFunctions involves a fundamental distinction between two stages of processing:

- A setup function within a nimbleFunction gives the steps that need to happen only once for each new situation (e.g., for each new model). Typically such steps include inspecting the model's variables and their relationships, such as determining which parts of a model will need to be calculated for a MCMC sampler. Setup functions are executed in R and never compiled.
- 2. One or more run functions within a nimbleFunction give steps that need to happen multiple times using the results of the setup function, such as the iterations of a MCMC sampler. Formally, run code is written in the NIMBLE language, which you can think of as a small subset of R along with features for operating models and related data structures. The NIMBLE language is what the NIMBLE compiler can automatically turn into C++ as part of a compiled nimbleFunction.

What NIMBLE does with a nimbleFunction is similar to what it does with a BUGS model:

¹or multiple model objects

²That's why it's easy to use new functions and distributions written as nimbleFunctions in BUGS code.

- 1. NIMBLE creates a working R version of the nimbleFunction. This is most useful for debugging (Section 15.7).
- 2. When you are ready, NIMBLE can generate C++ code, compile it, load it back into R and give you new objects that use the compiled C++ internally. Again, we refer to these steps all together as 'compilation'. The behavior of compiled nimbleFunctions is usually very similar, but not identical, to their uncompiled counterparts.

If you are familiar with object-oriented programming, you can think of a nimbleFunction as a class definition. The setup function initializes a new object and run functions are class methods. Member data are determined automatically as the objects from a setup function needed in run functions. If no setup function is provided, the nimbleFunction corresponds to a simple (compilable) function rather than a class.

More about writing algorithms is in Chapter 15.

3.3 The NIMBLE algorithm library

In Version 0.13.2, the NIMBLE algorithm library includes:

- 1. MCMC with samplers including conjugate (Gibbs), slice, adaptive random walk (with options for reflection or sampling on a log scale), adaptive block random walk, and elliptical slice, among others. You can modify sampler choices and configurations from R before compiling the MCMC. You can also write new samplers as nimbleFunctions.
- 2. Reversible jump MCMC for variable selection.
- 3. WAIC calculation for model comparison after an MCMC algorithm has been run.
- 4. A set of particle filter (sequential Monte Carlo) methods including a basic bootstrap filter, auxiliary particle filter, ensemble Kalman Filter, iterated filtering 2 filter (IF2), and Liu-West filter.
- 5. An ascent-based Monte Carlo Expectation Maximization (MCEM) algorithm.
- 6. A variety of basic functions that can be used as programming tools for larger algorithms. These include:
 - a. A likelihood function for arbitrary parts of any model.
 - b. Functions to simulate one or many sets of values for arbitrary parts of any model.
 - c. Functions to calculate the summed log probability (density) for one or many sets of values for arbitrary parts of any model along with stochastic dependencies in the model structure.

More about the NIMBLE algorithm library is in Chapter 8.

Chapter 4

Installing NIMBLE

4.1 Requirements to run NIMBLE

You can run NIMBLE on any of the three common operating systems: Linux, MacOS, or Windows. The following are required to run NIMBLE.

- 1. R, of course.
- 2. The igraph, coda and R6 R packages.
- 3. A working C++ compiler that NIMBLE can use from R on your system. There are standard open-source C++ compilers that the R community has already made easy to install. See Section 4.2 for instructions. You don't need to know anything about C++ to use NIMBLE. This must be done before installing NIMBLE.

NIMBLE also uses a couple of C++ libraries that you don't need to install, as they will already be on your system or are provided by NIMBLE.

- 1. The Eigen C++ library for linear algebra. This comes with NIMBLE, or you can use your own copy.
- 2. The BLAS and LAPACK numerical libraries. These come with R, but see Section 4.5.3 for how to use a faster version of the BLAS.

Most fairly recent versions of these requirements should work.

4.2 Installing a C++ compiler for NIMBLE to use

NIMBLE needs a C++ compiler and the standard utility make in order to generate and compile C++ for models and algorithms.¹

4.2.1 MacOS

On MacOS, you should install *Xcode*. The command-line tools, which are available as a smaller installation, should be sufficient. This is freely available from the Apple developer site and the App

¹This differs from most packages, which might need a C++ compiler only when the package is built. If you normally install R packages using install.packages on Windows or MacOS, the package arrives already built to your system.

Store.

In the somewhat unlikely event you want to install from the source package rather than the CRAN binary package, the easiest approach is to use the source package provided at R-nimble.org. If you do want to install from the source package provided by CRAN, you'll need to install this gfortran package.

4.2.2 Linux

On Linux, you can install the GNU compiler suite (gcc/g++). You can use the package manager to install pre-built binaries. On Ubuntu, the following command will install or update make, gcc and libc.

sudo apt-get install build-essential

4.2.3 Windows

On Windows, you should download and install Rtools.exe available from https://cran.r-project.org/bin/windows/Rtools/. Select the appropriate executable corresponding to your version of R (and follow the urge to update your version of R if you notice it is not the most recent).

Important: You must set the path so that the installer will add the location of the C++ compiler and related tools to your system's PATH variable, ensuring that R can find them. For R version 4.0 or greater (Rtools40) be sure to follow the instructions in the section Putting Rtools on the PATH. For R version 3.6.3 or lesser (i.e., using Rtools35.exe) make sure to check the box labelled "Add rtools to system PATH" (page 5 of the installation pages) (it should be checked by default). After you click 'Next', you will get a page with a window for customizing the new PATH variable. You shouldn't need to do anything there, so you can simply click 'Next' again.

4.3 Installing the NIMBLE package

Since NIMBLE is an R package, you can install it in the usual way, via install.packages("nimble") in R or using the R CMD INSTALL method if you download the package source directly.

NIMBLE can also be obtained from the NIMBLE website. To install from our website, please see our Download page for the specific invocation of install.packages.

4.4 Troubleshooting installation problems

We have tested the installation on the three commonly used platforms – MacOS, Linux, Windows². We don't anticipate problems with installation, but we want to hear about any and help resolve them.

The following are some troubleshooting tips that have helped users in some situations.

For Windows:

• Be sure you have set PATH when installing Rtools (see instructions above in the Windows installation section). Alternatively, one can set the PATH manually using syntax similar to

²We've tested NIMBLE on Windows 7, 8 and 10.

this (after changing C:\\Rtools\\bin;C:\\Rtools\\mingw_64\\bin to be appropriate for your system):

- Be sure the Rtools.exe version matches the R version.
- Try re-installing Rtools followed by re-installing NIMBLE.
- If you're using Rtools40, make sure you are using a newer version or RStudio (at least 1.2.5042).
- If there are filesystem or permissions issues, it is possible to install NIMBLE in a local directory using the lib argument to install.packages.
- In the past we've heard reports from Windows users of problems when their filesystem involved a space in a directory name in the path to RHOME. We think this problem has been resolved.
- We've also heard reports from Windows users of problems when R is installed on a network drive. A work-around is to install locally in a directory on a drive physically on the machine.

For MacOS:

- Newly installed Xcode/command line tools may need to be started once manually to provide a one-time permission before they will work from NIMBLE.
- Upgrading your MacOS version may result in an error while installing or using NIMBLE. Please try running the following in the Terminal to reinstall Xcode/command line tools:

```
xcode-select --install
```

• If multiple C++ compilers are present on a system, be sure the PATH will find the right one.

All operating systems:

• If problems arise from generating and compiling C++ files from the default location in R's tempdir(), one can use the dirName argument to compileNimble to put such files elsewhere, such as in a local working directory.

If those suggestions don't help, please post about installation problems to the nimble-users Google group or email nimble.stats@gmail.com.

4.5 Customizing your installation

For most installations, you can ignore low-level details. However, there are some options that some users may want to utilize.

4.5.1 Using your own copy of Eigen

NIMBLE uses the Eigen C++ template library for linear algebra. Version 3.2.1 of Eigen is included in the NIMBLE package and that version will be used unless the package's configuration script finds

another version on the machine. This works well, and the following is only relevant if you want to use a different (e.g., newer) version.

The configuration script looks in the standard include directories, e.g. /usr/include and /usr/local/include for the header file Eigen/Dense. You can specify a particular location in either of two ways:

- 1. Set the environment variable EIGEN_DIR before installing the R package, for example: export EIGEN_DIR=/usr/include/eigen3 in the bash shell.
- 2. Use

```
R CMD INSTALL --configure-args='--with-eigen=/path/to/eigen' nimble_VERSION.tar.gz

or

install.packages("nimble", configure.args = "--with-eigen=/path/to/eigen")
```

In these cases, the directory should be the full path to the directory that contains the Eigen directory, e.g., /usr/include/eigen3. It is not the full path to the Eigen directory itself, i.e., NOT /usr/include/eigen3/Eigen.

4.5.2 Using libnimble

NIMBLE generates specialized C++ code for user-specified models and nimbleFunctions. This code uses some NIMBLE C++ library classes and functions. By default, on Linux the library code is compiled once as a linkable library - libnimble.so. This single instance of the library is then linked with the code for each generated model. In contrast, the default for Windows and MacOS is to compile the library code as a static library - libnimble.a - that is compiled into each model's and each algorithm's own dynamically loadable library (DLL). This does repeat the same code across models and so occupies more memory. There may be a marginal speed advantage. If one would like to enable the linkable library in place of the static library (do this only on MacOS and other UNIX variants and not on Windows), one can install the source package with the configuration argument --enable-dylib set to true. First obtain the NIMBLE source package (which will have the extension .tar.gz from our website and then install as follows, replacing VERSION with the appropriate version number:

```
R CMD INSTALL --configure-args='--enable-dylib=true' nimble_VERSION.tar.gz
```

4.5.3 BLAS and LAPACK

NIMBLE also uses BLAS and LAPACK for some of its linear algebra (in particular calculating density values and generating random samples from multivariate distributions). NIMBLE will use the same BLAS and LAPACK installed on your system that R uses. Note that a fast (and where appropriate, threaded) BLAS can greatly increase the speed of linear algebra calculations. See Section A.3.1 of the R Installation and Administration manual available on CRAN for more details on providing a fast BLAS for your R installation.

4.5.4 Customizing compilation of the NIMBLE-generated C++

For each model or nimbleFunction, NIMBLE can generate and compile C++. To compile generated C++, NIMBLE makes system calls starting with R CMD SHLIB and therefore uses the regular R

configuration in \${R_HOME}/etc/\${R_ARCH}/Makeconf. NIMBLE places a Makevars file in the directory in which the code is generated, and R CMD SHLIB uses this file as usual.

In all but specialized cases, the general compilation mechanism will suffice. However, one can customize this. One can specify the location of an alternative Makevars (or Makevars.win) file to use. Such an alternative file should define the variables PKG_CPPFLAGS and PKG_LIBS. These should contain, respectively, the pre-processor flag to locate the NIMBLE include directory, and the necessary libraries to link against (and their location as necessary), e.g., Rlapack and Rblas on Windows, and libnimble. Advanced users can also change their default compilers by editing the Makevars file, see Section 1.2.1 of the Writing R Extensions manual available on CRAN.

Use of this file allows users to specify additional compilation and linking flags. See the Writing R Extensions manual for more details of how this can be used and what it can contain.

Part II Models in NIMBLE

Chapter 5

Writing models in NIMBLE's dialect of BUGS

Models in NIMBLE are written using a variation on the BUGS language. From BUGS code, NIMBLE creates a model object. This chapter describes NIMBLE's version of BUGS. The next chapter explains how to build and manipulate model objects.

5.1 Comparison to BUGS dialects supported by WinBUGS, OpenBUGS and JAGS

Many users will come to NIMBLE with some familiarity with WinBUGS, OpenBUGS, or JAGS, so we start by summarizing how NIMBLE is similar to and different from those before documenting NIMBLE's version of BUGS more completely. In general, NIMBLE aims to be compatible with the original BUGS language and also JAGS' version. However, at this point, there are some features not supported by NIMBLE, and there are some extensions that are planned but not implemented.

5.1.1 Supported features of BUGS and JAGS

- 1. Stochastic and deterministic¹ node declarations.
- 2. Most univariate and multivariate distributions.
- 3. Link functions.
- 4. Most mathematical functions.
- 5. 'for' loops for iterative declarations.
- 6. Arrays of nodes up to 4 dimensions.
- 7. Truncation and censoring as in JAGS using the T() notation and dinterval.

5.1.2 NIMBLE's Extensions to BUGS and JAGS

NIMBLE extends the BUGS language in the following ways:

1. User-defined functions and distributions – written as nimbleFunctions – can be used in model code. See Chapter 12.

¹NIMBLE calls non-stochastic nodes 'deterministic', whereas BUGS calls them 'logical'. NIMBLE uses 'logical' in the way R does, to refer to boolean (TRUE/FALSE) variables.

- 2. Multiple parameterizations for distributions, similar to those in R, can be used.
- 3. Named parameters for distributions and functions, similar to R function calls, can be used.
- 4. Linear algebra, including for vectorized calculations of simple algebra, can be used in deterministic declarations.
- 5. Distribution parameters can be expressions, as in JAGS but not in WinBUGS. Caveat: parameters to *multivariate* distributions (e.g., dmnorm) cannot be expressions (but an expression can be defined in a separate deterministic expression and the resulting variable then used).
- 6. Alternative models can be defined from the same model code by using if-then-else statements that are evaluated when the model is defined.
- 7. More flexible indexing of vector nodes within larger variables is allowed. For example one can place a multivariate normal vector arbitrarily within a higher-dimensional object, not just in the last index.
- 8. More general constraints can be declared using dconstraint, which extends the concept of JAGS' dinterval.
- 9. Link functions can be used in stochastic, as well as deterministic, declarations.²
- 10. Data values can be reset, and which parts of a model are flagged as data can be changed, allowing one model to be used for different data sets without rebuilding the model each time.
- 11. As of Version 0.6-6 we now support stochastic/dynamic indexes. More specifically in earlier versions all indexes needed to be constants. Now indexes can be other nodes or functions of other nodes. For a given dimension of a node being indexed, if the index is not constant, it must be a scalar value. So expressions such as mu[k[i], 3] or mu[k[i], 1:3] or mu[k[i], j[i]] are allowed, but not mu[k[i]:(k[i]+1)]. Nested dynamic indexes such as mu[k[j[i]]] are also allowed.

5.1.3 Not-yet-supported features of BUGS and JAGS

In this release, the following are not supported.

- 1. The appearance of the same node on the left-hand side of both a \leftarrow and a \sim declaration (used in WinBUGS for data assignment for the value of a stochastic node).
- 2. Multivariate nodes must appear with brackets, even if they are empty. E.g., x cannot be multivariate but x[] or x[2:5] can be.
- 3. NIMBLE generally determines the dimensionality and sizes of variables from the BUGS code. However, when a variable appears with blank indices, such as in x.sum <- sum(x[]), and if the dimensions of the variable are not clearly defined in other declarations, NIMBLE currently requires that the dimensions of x be provided when the model object is created (via nimbleModel).

5.2 Writing models

Here we introduce NIMBLE's version of BUGS. The WinBUGS, OpenBUGS and JAGS manuals are also useful resources for writing BUGS models, including many examples.

5.2.1 Declaring stochastic and deterministic nodes

BUGS is a declarative language for graphical (or hierarchical) models. Most programming languages are imperative, which means a series of commands will be executed in the order they are written. A

²But beware of the possibility of needing to set values for 'lifted' nodes created by NIMBLE.

declarative language like BUGS is more like building a machine before using it. Each line declares that a component should be plugged into the machine, but it doesn't matter in what order they are declared as long as all the right components are plugged in by the end of the code.

The machine in this case is a graphical model³. A *node* (sometimes called a vertex) holds one value, which may be a scalar or a vector. Edges define the relationships between nodes. A huge variety of statistical models can be thought of as graphs.

Here is the code to define and create a simple linear regression model with four observations.

```
library(nimble)
mc <- nimbleCode({</pre>
    intercept \sim dnorm(0, sd = 1000)
    slope \sim dnorm(0, sd = 1000)
    sigma ~ dunif(0, 100)
    for(i in 1:4) {
        predicted.y[i] <- intercept + slope * x[i]</pre>
        y[i] ~ dnorm(predicted.y[i], sd = sigma)
    }
})
model <- nimbleModel(mc, data = list(y = rnorm(4)))</pre>
library(igraph)
layout <- matrix(ncol = 2, byrow = TRUE,</pre>
   # These seem to be rescaled to fit in the plot area,
   # so I'll just use 0-100 as the scale
                  data = c(33, 100,
                            66, 100,
                            50, 0, # first three are parameters
                            15, 50, 35, 50, 55, 50, 75, 50, \# x's
                            20, 75, 40, 75, 60, 75, 80, 75, # predicted.y's
                            25, 25, 45, 25, 65, 25, 85, 25) # y's
                  )
sizes \leftarrow c(45, 30, 30,
            rep(20, 4),
            rep(50, 4),
            rep(20, 4))
edge.color <- "black"
    # c(
    # rep("green", 8),
    # rep("red", 4),
    # rep("blue", 4),
    # rep("purple", 4))
stoch.color <- "deepskyblue2"</pre>
```

³Technically, a directed acyclic graph

```
det.color <- "orchid3"</pre>
rhs.color <- "gray73"
fill.color <- c(
    rep(stoch.color, 3),
    rep(rhs.color, 4),
    rep(det.color, 4),
    rep(stoch.color, 4)
)
plot(model$graph, vertex.shape = "crectangle",
     vertex.size = sizes,
     vertex.size2 = 20,
     layout = layout,
     vertex.label.cex = 3.0,
     vertex.color = fill.color,
     edge.width = 3,
     asp = 0.5,
     edge.color = edge.color)
```

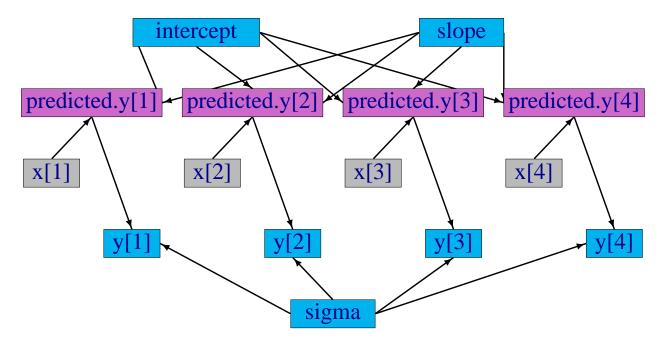


Figure 5.1: Graph of a linear regression model

The graph representing the model is shown in Figure 5.1. Each observation, y[i], is a node whose edges say that it follows a normal distribution depending on a predicted value, predicted.y[i], and standard deviation, sigma, which are each nodes. Each predicted value is a node whose edges say how it is calculated from slope, intercept, and one value of an explanatory variable, x[i], which are each nodes.

This graph is created from the following BUGS code:

```
{
   intercept ~ dnorm(0, sd = 1000)
   slope ~ dnorm(0, sd = 1000)
   sigma ~ dunif(0, 100)
   for(i in 1:4) {
      predicted.y[i] <- intercept + slope * x[i]
      y[i] ~ dnorm(predicted.y[i], sd = sigma)
   }
}</pre>
```

In this code, stochastic relationships are declared with '~' and deterministic relationships are declared with '<-'. For example, each y[i] follows a normal distribution with mean predicted.y[i] and standard deviation sigma. Each predicted.y[i] is the result of intercept + slope * x[i]. The for-loop yields the equivalent of writing four lines of code, each with a different value of i. It does not matter in what order the nodes are declared. Imagine that each line of code draws part of Figure 5.1, and all that matters is that the everything gets drawn in the end. Available distributions, default and alternative parameterizations, and functions are listed in Section 5.2.4.

An equivalent graph can be created by this BUGS code:

```
{
   intercept ~ dnorm(0, sd = 1000)
   slope ~ dnorm(0, sd = 1000)
   sigma ~ dunif(0, 100)
   for(i in 1:4) {
      y[i] ~ dnorm(intercept + slope * x[i], sd = sigma)
   }
}
```

In this case, the predicted.y[i] nodes in Figure 5.1 will be created automatically by NIMBLE and will have a different name, generated by NIMBLE.

5.2.2 More kinds of BUGS declarations

Here are some examples of valid lines of BUGS code. This code does not describe a sensible or complete model, and it includes some arbitrary indices (e.g. mvx[8:10, i]) to illustrate flexibility. Instead the purpose of each line is to illustrate a feature of NIMBLE's version of BUGS.

```
# 1. normal distribution with BUGS parameter order
x ~ dnorm(a + b * c, tau)
# 2. normal distribution with a named parameter
y ~ dnorm(a + b * c, sd = sigma)
# 3. For-loop and nested indexing
for(i in 1:N) {
    for(j in 1:M[i]) {
        z[i,j] ~ dexp(r[ blockID[i] ])
    }
}
# 4. multivariate distribution with arbitrary indexing
```

When a variable appears only on the right-hand side, it can be provided via constants (in which case it can never be changed) or via data or inits, as discussed in Chapter 6.

Notes on the comment-numbered lines are:

- 1. x follows a normal distribution with mean a + b*c and precision tau (default BUGS second parameter for dnorm).
- 2. y follows a normal distribution with the same mean as x but a named standard deviation parameter instead of a precision parameter (sd = 1/sqrt(precision)).
- 3. z[i, j] follows an exponential distribution with parameter r[blockID[i]]. This shows how for-loops can be used for indexing of variables containing multiple nodes. Variables that define for-loop indices (N and M) must also be provided as constants.
- 4. The arbitrary block mvx[8:10, i] follows a multivariate normal distribution, with a named covariance matrix instead of BUGS' default of a precision matrix. As in R, curly braces for for-loop contents are only needed if there is more than one line.
- 5. w follows a user-defined distribution. See Chapter 12.
- 6. d1 is a scalar deterministic node that, when calculated, will be set to a + b.
- 7. d2 is a vector deterministic node using matrix multiplication in R's syntax.
- 8. d3 is a deterministic node using a user-provided function. See Chapter 12.

5.2.2.1 More about indexing

Examples of allowed indexing include:

- x[i] # a single index
- x[i:j] # a range of indices
- x[i:j,k:1] # multiple single indices or ranges for higher-dimensional arrays
- x[i:j,] # blank indices indicating the full range
- x[3*i+7] # computed indices
- x[(3*i):(5*i+1)] # computed lower and upper ends of an index range
- x[k[i]+1] # a dynamic (and computed) index
- x[k[j[i]]] # nested dynamic indexes
- x[k[i], 1:3] # nested indexing of rows or columns

NIMBLE does not allow multivariate nodes to be used without square brackets, which is an incompatibility with JAGS. Therefore a statement like xbar <- mean(x) in JAGS must be converted to

xbar <- mean(x[]) (if x is a vector) or xbar <- mean(x[,]) (if x is a matrix) for NIMBLE⁴. Section 6.1.1.5 discusses how to provide NIMBLE with dimensions of x when needed.

Generally NIMBLE supports R-like linear algebra expressions and attempts to follow the same rules as R about dimensions (although in some cases this is not possible). For example, x[1:3] %*% y[1:3] converts x[1:3] into a row vector and thus computes the inner product, which is returned as a 1×1 matrix (use inprod to get it as a scalar, which it typically easier). Like in R, a scalar index will result in dropping a dimension unless the argument drop=FALSE is provided. For example, mymatrix[i, 1:3] will be a vector of length 3, but mymatrix[i, 1:3, drop=FALSE] will be a 1×3 matrix. More about indexing and dimensions is discussed in Section 11.3.2.7.

5.2.3 Vectorized versus scalar declarations

Suppose you need nodes logY[i] that should be the log of the corresponding Y[i], say for i from 1 to 10. Conventionally this would be created with a for loop:

```
{
    for(i in 1:10) {
        logY[i] <- log(Y[i])
    }
}</pre>
```

Since NIMBLE supports R-like algebraic expressions, an alternative in NIMBLE's dialect of BUGS is to use a vectorized declaration like this:

```
{
   logY[1:10] <- log(Y[1:10])
}</pre>
```

There is an important difference between the models that are created by the above two methods. The first creates 10 scalar nodes, logY[1],..., logY[10]. The second creates one vector node, logY[1:10]. If each logY[i] is used separately by an algorithm, it may be more efficient computationally if they are declared as scalars. If they are all used together, it will often make sense to declare them as a vector.

5.2.4 Available distributions

5.2.4.1 Distributions

NIMBLE supports most of the distributions allowed in BUGS and JAGS. Table 5.1 lists the distributions that are currently supported, with their default parameterizations, which match those of BUGS⁵. NIMBLE also allows one to use alternative parameterizations for a variety of distributions as described next. See Section 12.2 to learn how to write new distributions using nimbleFunctions.

⁴In nimbleFunctions, as explained in later chapters, square brackets with blank indices are not necessary for multivariate objects.

⁵Note that the same distributions are available for writing nimbleFunctions, but in that case the default parameterizations and function names match R's when possible. Please see Section 11.2.4 for how to use distributions in nimbleFunctions.

Table 5.1: Distributions with their default order of parameters. The value of the random variable is denoted by x.

Name	Usage	Density	Lower	Upp
Bernoulli	$\begin{array}{l} \texttt{dbern(prob = p)} \\ 0$	$p^x(1-p)^{1-x}$	0	1
Beta	dbeta(shape1 = a,	$\frac{x^{a-1}(1-x)^{b-1}}{\beta(a,b)}$	0	1
Binomial	shape2 = b), $a > 0$, $b > 0$ dbin(prob = p, size = n) $0 , n \in \mathbb{N}^*$	$\binom{n}{x}p^x(1-p)^{n-x}$	0	n
CAR (intrinsic)	dcar_normal(adj, weights, num, tau, c, zero_mean)	see chapter 9 for details		
CAR (proper)	<pre>dcar_proper(mu, C, adj, num, M, tau, gamma)</pre>	see chapter 9 for details		
Categorical	$ ext{dcat(prob = p)} \ p \in (\mathbb{R}^+)^N$	$rac{p_x}{\sum_i p_i}$	1	N
Chi-square	${\tt dchisq(df = k)}, k>0$	$\frac{x^{\frac{k}{2}-1}\exp(-x/2)}{2^{\frac{k}{2}}\Gamma(\frac{k}{2})}$	0	
Double exponential	$\label{eq:dexp} \begin{split} \mathrm{ddexp}(\mathrm{location} &= \mu, \\ \mathrm{rate} &= \tau) \text{,} \tau > 0 \end{split}$	$\frac{\tau}{2}\exp(-\tau x-\mu)$		
Dirichlet	$ddirch(alpha = \alpha)$	$\Gamma(\sum_j \alpha_j) \prod_j rac{x_j^{lpha_j-1}}{\Gamma(lpha_j)}$	0	
Exponential Flat	$\alpha_j \geq 0$ dexp(rate = λ), $\lambda > 0$ dflat()	$\lambda \exp(-\lambda x)$ $\propto 1 \text{ (improper)}$	0	
Gamma	dgamma(shape = r , rate = λ) $\lambda > 0, r > 0$	$\frac{\lambda^r x^{r-1} \exp(-\lambda x)}{\Gamma(r)}$	0	
Half flat	dhalfflat()	$\propto 1 \text{ (improper)}$	0	
Inverse gamma	dinvgamma(shape = r, scale = λ) $\lambda > 0, r > 0$	$\frac{\lambda^r x^{-(r+1)} \exp(-\lambda/x)}{\Gamma(r)}$	0	
LKJ Correl'n Cholesky	dlkj_corr_cholesky(eta = η , size = p), $\eta > 0$	$\prod_{i=2}^{p} x_{ii}^{p-i+2\eta-2}$		
Logistic	dlogis(location = μ ,	$\frac{\tau \exp\{(x-\mu)\tau\}}{\left[1+\exp\{(x-\mu)\tau\}\right]^2}$		
Log-normal	rate = τ), $\tau > 0$ dlnorm(meanlog = μ , taulog = τ), $\tau > 0$	$\left(\tfrac{\tau}{2\pi} \right)^{\frac{1}{2}} x^{-1} \exp \left\{ -\tau (\log(x) - \mu)^2 / 2 \right\}$	0	
Multinomial	dmulti(prob = p, size = n) $\sum_{i} x_{i} = n$	$n!\prod_j rac{p_j^{x_j}}{x_j!}$		
Multivariate normal	dmnorm(mean = μ , prec = Λ) Λ positive definite	$(2\pi)^{-\frac{d}{2}} \Lambda ^{\frac{1}{2}}\exp\{-\frac{(x-\mu)^T\Lambda(x-\mu)}{2}\}$		
Multivariate	${\tt dmvt(mu = \mu, prec = \Lambda``}$	$\frac{\Gamma(\frac{\nu+d}{2})}{\Gamma(\frac{\nu}{2})(\nu\pi)^{d/2}} \Lambda ^{1/2}(1+\frac{(x-\mu)^T\Lambda(x-\mu)}{\nu})^{-\frac{\nu+d}{2}}$		
Student t Negative binomial	df = ν), Λ positive def. dnegbin(prob = p, size = r) 0	$\binom{x+r-1}{x}p^r(1-p)^x$	0	

Name	Usage	Density	Lower	Upper
Normal	$\begin{array}{l} \texttt{dnorm(mean = }\mu\text{, tau = }\tau\text{)} \\ \tau > 0 \end{array}$	$\left(\frac{\tau}{2\pi}\right)^{\frac{1}{2}} \exp\{-\tau(x-\mu)^2/2\}$		
Poisson	$\begin{array}{l} {\rm dpois}({\rm lambda} = \lambda) \\ \lambda > 0 \end{array}$	$\frac{\exp(-\lambda)\lambda^x}{x!}$	0	
Student t	$dt(mu = \mu, tau = \tau, df = k)$ $\tau > 0, k > 0$	$\frac{\Gamma(\frac{k+1}{2})}{\Gamma(\frac{k}{2})} \left(\frac{\tau}{k\pi}\right)^{\frac{1}{2}} \left\{1 + \frac{\tau(x-\mu)^2}{k}\right\}^{-\frac{(k+1)}{2}}$		
Uniform	dunif(min = a, max = b) $a < b$	$\frac{1}{b-a}$	a	b
Weibull	dweib(shape = v, lambda = λ) $v > 0, \lambda > 0$	$v\lambda x^{v-1}\exp(-\lambda x^v)$	0	
Wishart	$\begin{array}{l} \texttt{dwish}(\texttt{R} = \texttt{R}, \ \texttt{df} = \texttt{k}) \\ \texttt{R} \ p \times p \ \text{pos. def.}, \ k \geq p \end{array}$	$\frac{ x ^{(k-p-1)/2} R ^{k/2}\exp\{-\operatorname{tr}(Rx)/2\}}{2^{pk/2}\pi^{p(p-1)/4}\prod_{i=1}^{p}\Gamma((k+1-i)/2)}$		
Inverse	dinvwish(S = S, df = k)	$\frac{ x ^{-(k+p+1)/2} S ^{k/2}\exp\{-\operatorname{tr}(Sx^{-1})/2\}}{2^{pk/2}\pi^{p(p-1)/4}\prod_{i=1}^{p}\Gamma((k+1-i)/2)}$		
Wishart	S $p \times p$ pos. def., $k \ge p$	$11_{i=1} 1 \left((\kappa + 1 - t)/2 \right)$		

5.2.4.1.1 Improper distributions Note that dcar_normal, dflat and dhalfflat specify improper prior distributions and should only be used when the posterior distribution of the model is known to be proper. Also for these distributions, the density function returns the unnormalized density and the simulation function returns NaN so these distributions are not appropriate for algorithms that need to simulate from the prior or require proper (normalized) densities.

5.2.4.1.2 LKJ distribution for correlation matrices NIMBLE provides the LKJ distribution via dlkj_corr_cholesky for correlation matrices, discussed in Section 24 of Stan Development Team (2021a) and based on Lewandowski et al. (2009). This distribution has various advantages (both from a modeling perspective and an MCMC fitting perspective) over other prior distributions for correlation or covariance matrices such as the inverse Wishart distribution.

For computational efficiency, the distribution is on the Cholesky factor of the correlation matrix rather than the correlation matrix itself. As a result using this distribution in a model involves a bit of care, including efficiently creating a covariance matrix from the correlation matrix and a set of standard deviations.

Here is some example code illustrating how to use the distribution in a model.

```
code <- nimbleCode({
    Ustar[1:p,1:p] ~ dlkj_corr_cholesky(1.3, p)
    U[1:p,1:p] <- uppertri_mult_diag(Ustar[1:p, 1:p], sds[1:p])
    for(i in 1:n)
        y[i, 1:p] ~ dmnorm(mu[1:p], cholesky = U[1:p, 1:p], prec_param = 0)
})</pre>
```

Note that we need to take the upper triangular Cholesky factor (Ustar) of the correlation matrix and convert it to the upper triangular Cholesky factor of a covariance matrix (U), based on a vector of standard deviations (sds), and use that in the parameterization of dmnorm that directly (and therefore efficiently) uses the Cholesky of the covariance.

Other approaches are possible, but unless care is taken they are likely to be less computationally efficient than the template above.

The template uses a user-defined nimbleFunction to efficiently combine the Cholesky of the correlation matrix with a vector of standard deviations to produce the Cholesky of the covariance matrix. That function is given here:

```
uppertri_mult_diag <- nimbleFunction(
    run = function(mat = double(2), vec = double(1)) {
        returnType(double(2))
        p <- length(vec)
        out <- matrix(nrow = p, ncol = p, init = FALSE)
        for(i in 1:p)
            out[ , i] <- mat[ , i] * vec[i]
        return(out)
    })</pre>
```

5.2.4.2 Alternative parameterizations for distributions

NIMBLE allows one to specify distributions in model code using a variety of parameterizations, including the BUGS parameterizations. Available parameterizations are listed in Table 5.2. To understand how NIMBLE handles alternative parameterizations, it is useful to distinguish three cases, using the gamma distribution as an example:

- 1. A canonical parameterization is used directly for computations⁶. For gamma, this is (shape, scale).
- 2. The BUGS parameterization is the one defined in the original BUGS language. In general this is the parameterization for which conjugate MCMC samplers can be executed most efficiently. For dgamma, this is (shape, rate).
- 3. An alternative parameterization is one that must be converted into the canonical parameterization. For dgamma, NIMBLE provides both (shape, rate) and (mean, sd) parameterization and creates nodes to calculate (shape, scale) from either (shape, rate) or (mean, sd). In the case of dgamma, the BUGS parameterization is also an alternative parameterization.

Since NIMBLE provides compatibility with existing BUGS and JAGS code, the order of parameters places the BUGS parameterization first. For example, the order of parameters for dgamma is dgamma(shape, rate, scale, mean, sd). Like R, if parameter names are not given, they are taken in order, so that (shape, rate) is the default. This happens to match R's order of parameters, but it need not. If names are given, they can be given in any order. NIMBLE knows that rate is an alternative to scale and that (mean, sd) are an alternative to (shape, scale or rate).

⁶Usually this is the parameterization in the Rmath header of R's C implementation of distributions.

Table 5.2: Distribution parameterizations allowed in NIM-BLE. The first column indicates the supported parameterizations for distributions given in Table 5.1. The second column indicates the relationship to the *canonical* parameterization used in NIMBLE.

Parameterization	NIMBLE re-parameterization
dbern(prob)	dbin(size = 1, prob)
dbeta(shape1, shape2)	canonical
dbeta(mean, sd)	dbeta(shape1 = mean^2 * (1-mean) / sd^2 - mean, shape2 = mean * (1 - mean)^2 / sd^2 + mean - 1)
dbin(prob, size)	canonical
dcat(prob)	canonical
dchisq(df)	canonical
<pre>ddexp(location, scale)</pre>	canonical
<pre>ddexp(location, rate)</pre>	<pre>ddexp(location, scale = 1 / rate)</pre>
<pre>ddexp(location, var)</pre>	<pre>ddexp(location, scale = sqrt(var / 2))</pre>
ddirch(alpha)	canonical
<pre>dexp(rate)</pre>	canonical
<pre>dexp(scale)</pre>	<pre>dexp(rate = 1/scale)</pre>
dgamma(shape, scale)	canonical
dgamma(shape, rate)	<pre>dgamma(shape, scale = 1 / rate)</pre>
dgamma(mean, sd)	dgamma(shape = mean^2/sd^2, scale = sd^2/mean)
dinvgamma(shape, rate)	canonical
<pre>dinvgamma(shape, scale)</pre>	dgamma(shape, rate = 1 / scale)
<pre>dlkj_corr_cholesky(eta)</pre>	canonical
<pre>dlogis(location, scale)</pre>	canonical
<pre>dlogis(location, rate)</pre>	<pre>dlogis(location, scale = 1 / rate)</pre>
<pre>dlnorm(meanlog, sdlog)</pre>	canonical
<pre>dlnorm(meanlog, taulog)</pre>	<pre>dlnorm(meanlog, sdlog = 1 / sqrt(taulog))</pre>
<pre>dlnorm(meanlog, varlog)</pre>	<pre>dlnorm(meanlog, sdlog = sqrt(varlog))</pre>
<pre>dmulti(prob, size)</pre>	canonical
<pre>dmnorm(mean, cholesky,</pre>	canonical (precision)
prec_param=1)	
<pre>dmnorm(mean, cholesky,</pre>	canonical (covariance)
prec_param=0)	
<pre>dmnorm(mean, prec)</pre>	<pre>dmnorm(mean, cholesky = chol(prec), prec_param=1)</pre>
dmnorm(mean, cov)	<pre>dmnorm(mean, cholesky = chol(cov), prec_param=0)</pre>
<pre>dmvt(mu, cholesky, df,</pre>	canonical (precision/inverse scale)
prec_param=1)	
<pre>dmvt(mu, cholesky, df,</pre>	canonical (scale)
prec_param=0)	
<pre>dmvt(mu, prec, df)</pre>	<pre>dmvt(mu, cholesky = chol(prec), df, prec_param=1)</pre>
<pre>dmvt(mu, scale, df)</pre>	<pre>dmvt(mu, cholesky = chol(scale), df, prec_param=0)</pre>
dlkj_corr_cholesky(canonical
shape, size)	
<pre>dnegbin(prob, size)</pre>	canonical

Parameterization	NIMBLE re-parameterization
dnorm(mean, sd)	canonical
<pre>dnorm(mean, tau)</pre>	<pre>dnorm(mean, sd = 1 / sqrt(var))</pre>
<pre>dnorm(mean, var)</pre>	<pre>dnorm(mean, sd = sqrt(var))</pre>
dpois(lambda)	canonical
dt(mu, sigma, df)	canonical
dt(mu, tau, df)	<pre>dt(mu, sigma = 1 / sqrt(tau), df)</pre>
dt(mu, sigma2, df)	<pre>dt(mu, sigma = sqrt(sigma2), df)</pre>
<pre>dunif(min, max)</pre>	canonical
dweib(shape, scale)	canonical
dweib(shape, rate)	<pre>dweib(shape, scale = 1 / rate)</pre>
dweib(shape, lambda)	<pre>dweib(shape, scale = lambda^(- 1 / shape))</pre>
dwish(cholesky, df,	canonical (scale)
scale_param=1)	
dwish(cholesky, df,	canonical (inverse scale)
scale_param=0)	
dwish(R, df)	<pre>dwish(cholesky = chol(R), df, scale_param = 0)</pre>
dwish(S, df)	<pre>dwish(cholesky = chol(S), df, scale_param = 1)</pre>
dinvwish(cholesky, df,	canonical (scale)
scale_param=1)	
dinvwish(cholesky, df,	canonical (inverse scale)
scale_param=0)	
dinvwish(R, df)	<pre>dinvwish(cholesky = chol(R), df, scale_param = 0)</pre>
dinvwish(S, df)	<pre>dinvwish(cholesky = chol(S), df, scale_param = 1)</pre>

Note that for multivariate normal, multivariate t, Wishart, and Inverse Wishart, the canonical parameterization uses the Cholesky decomposition of one of the precision/inverse scale or covariance/scale matrix. For example, for the multivariate normal, if prec_param=TRUE, the cholesky argument is treated as the Cholesky decomposition of a precision matrix. Otherwise it is treated as the Cholesky decomposition of a covariance matrix.

In addition, NIMBLE supports alternative distribution names, known as aliases, as in JAGS, as specified in Table 5.3.

Distribution Canonical name Alias

Table 5.3: Distributions with alternative names (aliases)

Distribution	Canonical name	Alias
Binomial	dbin	dbinom
Chi-square	dchisq	dchisqr
Double exponential	ddexp	dlaplace
Dirichlet	ddirch	$\operatorname{ddirich}$
Multinomial	dmulti	dmultinom
Negative binomial	dnegbin	dnbinom
Weibull	dweib	dweibull
Wishart	dwish	dwishart

We plan to, but do not currently, include the following distributions as part of core NIMBLE: beta-binomial, Dirichlet-multinomial, F, Pareto, or forms of the multivariate t other than the standard one provided.

5.2.5 Available BUGS language functions

Tables 5.4-5.5 show the available operators and functions. Support for more general R expressions is covered in Chapter 11 about programming with nimbleFunctions.

For the most part NIMBLE supports the functions used in BUGS and JAGS, with exceptions indicated in the table. Additional functions provided by NIMBLE are also listed. Note that we provide distribution functions for use in calculations, namely the 'p', 'q', and 'd' functions. See Section 11.2.4 for details on the syntax for using distribution functions as functions in deterministic calculations, as only some parameterizations are allowed and the names of some distributions differ from those used to define stochastic nodes in a model.

Table 5.4: Functions operating on scalars, many of which can operate on each element (component-wise) of vectors and matrices. Status column indicates if the function is currently provided in NIMBLE. Vector input column indicates if the function can take a vector as an argument (i.e., if the function is vectorized).

Usage	Description	Comments	Status	Vector input
x y, x & y	logical OR () and AND(&)		yes	yes
!x	logical not		yes	yes
x > y, $x >= y$	greater than (and or equal to)		yes	yes
$x < y$, $x \le y$	less than (and or equal to)		yes	yes
x != y, x == y	(not) equals		yes	yes
x + y, x - y, x * y	component-wise operators	mix of scalar and vector	yes	yes
x / y	component-wise division	vector x and scalar y	yes	yes
x^y, pow(x, y)	power	x^y ; vector x ,scalar y	yes	yes
х %% у	modulo (remainder)		yes	no
min(x1, x2),	min. (max.) of two scalars		yes	See pmin,
max(x1, x2)				pmax
exp(x)	exponential		yes	yes
log(x)	natural logarithm		yes	yes
sqrt(x)	square root		yes	yes
abs(x)	absolute value		yes	yes
step(x)	step function at 0	0 if $x < 0, 1$ if $x >= 0$	yes	yes
equals(x, y)	equality of two scalars	1 if $x == y$, 0 if $x! = y$	yes	yes
cube(x)	third power	x^3	yes	yes
sin(x), cos(x),	trigonometric functions		yes	yes
tan(x)				
asin(x), $acos(x)$,	inverse trigonometric functions		yes	yes
atan(x)				
asinh(x), $acosh(x)$,	inv. hyperbolic trig. functions		yes	yes
atanh(x)				

Usage	Description	Comments	Status	Vector input
logit(x)	logit	$\log(x/(1-x))$	yes	yes
<pre>ilogit(x), expit(x)</pre>	inverse logit	$\exp(x)/(1+\exp(x))$	yes	yes
<pre>probit(x)</pre>	probit (Gaussian quantile)	$\Phi^{-1}(x)$	yes	yes
<pre>iprobit(x), phi(x)</pre>	inverse probit (Gaussian CDF)	$\Phi(x)$	yes	yes
cloglog(x)	complementary log log	$\log(-\log(1-x))$	yes	yes
icloglog(x)	inverse complementary log log	$1 - \exp(-\exp(x))$	yes	yes
<pre>ceiling(x)</pre>	ceiling function	$\lceil (x) \rceil$	yes	yes
floor(x)	floor function	$\lfloor (x) \rfloor$	yes	yes
round(x)	round to integer		yes	yes
trunc(x)	truncation to integer		yes	yes
<pre>lgamma(x), loggam(x)</pre>	log gamma function	$\log \Gamma(x)$	yes	yes
besselK(k, nu,	modified bessel function	returns vector even if	yes	yes
\dots expon.scaled)	of the second kind	k a matrix/array		
log1p(x)	$\log \text{ of } 1 + x$	$\log(1+x)$	yes	yes
lfactorial(x),	log factorial	$\log x!$	yes	yes
logfact(x)				
qDIST(x, PARAMS)	"q" distribution functions	canonical parameterization	yes	yes
pDIST(x, PARAMS)	"p" distribution functions	canonical parameterization	yes	yes
rDIST(x, PARAMS)	"r" distribution functions	canonical parameterization	yes	yes
dDIST(x, PARAMS)	"d" distribution functions	canonical parameterization	yes	yes
sort(x)			no	
rank(x, s)			no	
<pre>ranked(x, s)</pre>			no	
order(x)			no	

Table 5.5: Functions operating on vectors and matrices. Status column indicates if the function is currently provided in NIMBLE.

Usage	Description	Comments	Status
inverse(x)	matrix inverse	x symmetric, positive def.	yes
chol(x)	matrix Cholesky factorization	x symmetric, positive def.	yes
		returns upper triang. matrix	
t(x)	matrix transpose	x^{\top}	yes
x%*%y	matrix multiply	xy; x, y conformant	yes
<pre>inprod(x, y)</pre>	dot product	$x^{\top}y$; x and y vectors	yes
solve(x,y)	solve system of equations	$x^{-1}y$; y matrix or vector	yes
forwardsolve(x, y)	solve lower-triangular system of equations	$x^{-1}y$; x lower-triangular	yes
<pre>backsolve(x, y)</pre>	solve upper-triangular system of equations	$x^{-1}y$; x upper-triangular	yes
logdet(x)	log matrix determinant	$\log x $	yes
asRow(x)	convert vector to 1-row matrix	sometimes automatic	yes
asCol(x)	convert vector to 1-column matrix	sometimes automatic	yes
sum(x)	sum of elements of x		yes
mean(x)	mean of elements of x		yes
sd(x)	standard deviation of elements of ${\tt x}$		yes

Usage	Description	Comments	Status
prod(x)	product of elements of x		yes
min(x), max(x)	min. (max.) of elements of x		yes
<pre>pmin(x, y), pmax(x,y)</pre>	vector of mins (maxs) of elements of		yes
	x and y		
<pre>interp.lin(x, v1, v2)</pre>	linear interpolation		no
eigen(x)\$values	matrix eigenvalues x symmetric		yes
eigen(x)\$vectors	matrix eigenvectors	x symmetric	yes
svd(x)\$d	matrix singular values		yes
svd(x)\$u	matrix left singular vectors		yes
svd(x)\$v	matrix right singular vectors		yes

See Section 12.1 to learn how to use nimbleFunctions to write new functions for use in BUGS code.

5.2.6 Available link functions

NIMBLE allows the link functions listed in Table 5.6.

Link function	Description	Range	Inverse
cloglog(y) <- x	Complementary log log	0 < y < 1	y <- icloglog(x)
log(y) <- x	Log	0 < y	$y \leftarrow exp(x)$
$logit(y) \leftarrow x$	Logit	0 < y < 1	y <- expit(x)
<pre>probit(y) <- x</pre>	Probit	0 < y < 1	<pre>y <- iprobit(x)</pre>

Table 5.6: Link functions.

Link functions are specified as functions applied to a node on the left hand side of a BUGS expression. To handle link functions in deterministic declarations, NIMBLE converts the declaration into an equivalent inverse declaration. For example, $log(y) \leftarrow x$ is converted into $y \leftarrow exp(x)$. In other words, the link function is just a simple variant for conceptual clarity.

To handle link functions in a stochastic declaration, NIMBLE does some processing that inserts an additional node into the model. For example, the declaration logit(p[i]) ~ dnorm(mu[i],1), is equivalent to the following two declarations:

- logit_p[i] ~ dnorm(mu[i], 1),
- p[i] <- expit(logit_p[i])

where expit is the inverse of logit.

Note that NIMBLE does not provide an automatic way of initializing the additional node (logit_p[i] in this case), which is a parent node of the explicit node (p[i]), without explicitly referring to the additional node by the name that NIMBLE generates.

5.2.7 Truncation, censoring, and constraints

NIMBLE provides three ways to declare boundaries on the value of a variable, each for different situations. We introduce these and comment on their relationships to related features of JAGS and

BUGS. The three methods are:

5.2.7.1 Truncation

Either of the following forms,

```
x \sim dnorm(0, sd = 10) T(0, a)

x \sim T(dnorm(0, sd = 10), 0, a)
```

declares that **x** follows a normal distribution between 0 and **a** (inclusive of 0 and **a**). Either boundary may be omitted or may be another node, such as **a** in this example. The first form is compatible with JAGS, but in NIMBLE it can only be used when reading code from a text file. When writing model code in R, the second version must be used.

Truncation means the possible values of x are limited a priori, hence the probability density of x must be normalized⁷. In this example it would be the normal probability density divided by its integral from 0 to a. Like JAGS, NIMBLE also provides I as a synonym for T to accommodate older BUGS code, but T is preferred because it disambiguates multiple usages of I in BUGS.

5.2.7.2 Censoring

Censoring refers to the situation where one datum gives the lower or upper bound on an unobserved random variable. This is common in survival analysis, when for an individual still surviving at the end of a study, the age of death is not known and hence is 'censored' (right-censoring). NIMBLE adopts JAGS syntax for censoring, as follows:

```
censored[i] ~ dinterval(t[i], c[i])
t[i] ~ dweib(r, mu[i])
```

In the case of right-censoring, censored[i] should be given as data with a value of 1 if t[i] is right-censored (t[i] > c[i]) and 0 if it is observed. The data vector for t should have NA (indicating missing data) for any censored t[i] entries. (As a result, these nodes will be sampled in an MCMC.) The vector for c should give the censoring times corresponding to censored entries and a value above the observed times for uncensored entries (e.g., Inf). The values for c can be provided via constants, inits, or data. We recommend providing via constants for values that are fixed and via inits for values that are to be estimated or might be changed for some reason.

Left-censored observations would be specified by setting censored[i] to 0 and t[i] to NA.

The dinterval is not really a distribution but rather a trick: in the above example when censored[i] = 1 it gives a 'probability' of 1 if t[i] > c[i] and 0 otherwise. This means that t[i] <= c[i] is treated as impossible. More generally than simple right- or left-censoring, censored[i] ~ dinterval(t[i], c[i,]) is defined such that for a vector of increasing cutpoints, c[i,], t[i] is enforced to fall within the censored[i]-th cutpoint interval. This is done by setting data censored[i] as follows:

⁷NIMBLE uses the CDF and inverse CDF (quantile) functions of a distribution to do this; in some cases if one uses truncation to include only the extreme tail of a distribution, numerical difficulties can arise.

(The i index is provided only for consistency with the previous example.) The most common uses of dinterval will be for left- and right-censored data, in which case c[i,] will be a single value (and typically given as simply c[i]), and for interval-censored data, in which case c[i,] will be a vector of two values.

Nodes following a dinterval distribution should normally be set as data with known values. Otherwise, the node may be simulated during initialization in some algorithms (e.g., MCMC) and thereby establish a permanent, perhaps unintended, constraint.

Censoring differs from truncation because censoring an observation involves bounds on a random variable that could have taken any value, while in truncation we know a priori that a datum could not have occurred outside the truncation range.

5.2.7.3 Constraints and ordering

NIMBLE provides a more general way to enforce constraints using dconstraint(cond). For example, we could specify that the sum of mu1 and mu2 must be positive like this:

```
mu1 ~ dnorm(0, 1)
mu2 ~ dnorm(0, 1)
constraint_data ~ dconstraint( mu1 + mu2 > 0 )
```

with constraint_data set (as data) to 1. This is equivalent to a half-normal distribution on the half-plane $\mu_1 + \mu_2 > 0$. Nodes following dconstraint should be provided as data for the same reason of avoiding unintended initialization described above for dinterval.

Formally, dconstraint(cond) is a probability distribution on $\{0,1\}$ such that P(1)=1 if cond is TRUE and P(0)=1 if cond is FALSE.

Of course, in many cases, parameterizing the model so that the constraints are automatically respected may be a better strategy than using dconstraint. One should be cautious about constraints that would make it hard for an MCMC or optimization to move through the parameter space (such as equality constraints that involve two or more parameters). For such restrictive constraints, general purpose algorithms that are not tailored to the constraints may fail or be inefficient. If constraints are used, it will generally be wise to ensure the model is initialized with values that satisfy them.

5.2.7.3.1 Ordering To specify an ordering of parameters, such as $\alpha_1 <= \alpha_2 <= \alpha_3$ one can use dconstraint as follows:

```
constraint_data ~ dconstraint( alpha1 <= alpha2 & alpha2 <= alpha3 )</pre>
```

Note that unlike in BUGS, one cannot specify prior ordering using syntax such as

```
alpha[1] ~ dnorm(0, 1) I(, alpha[2])
alpha[2] ~ dnorm(0, 1) I(alpha[1], alpha[3])
alpha[3] ~ dnorm(0, 1) I(alpha[2], )
```

as this does not represent a directed acyclic graph.

Also note that specifying prior ordering using T(,) can result in possibly unexpected results. For example:

```
alpha1 ~ dnorm(0, 1)
alpha2 ~ dnorm(0, 1) T(alpha1, )
alpha3 ~ dnorm(0, 1) T(alpha2, )
```

will enforce $alpha1 \le alpha2 \le alpha3$, but it does not treat the three parameters symmetrically. Instead it puts a marginal prior on alpha1 that is standard normal and then constrains alpha2 and alpha3 to follow truncated normal distributions. This is not equivalent to a symmetric prior on the three alphas that assigns zero probability density when values are not in order.

NIMBLE does not support the JAGS sort syntax.

Chapter 6

Building and using models

This chapter explains how to build and manipulate model objects starting from BUGS code.

6.1 Creating model objects

NIMBLE provides two functions for creating model objects: nimbleModel and readBUGSmodel. The first, nimbleModel, is more general and was illustrated in Chapter 2. The second, readBUGSmodel provides compatibility with BUGS file formats for models, variables, data, and initial values for MCMC.

In addition one can create new model objects from existing model objects.

6.1.1 Using *nimbleModel* to create a model

nimbleModel processes BUGS code to determine all the nodes, variables, and their relationships in a model. Any constants must be provided at this step. Data and initial values can optionally be provided. BUGS code passed to nimbleModel must go through nimbleCode.

We look again at the pump example from the introduction:

6.1.1.1 Data and constants

NIMBLE makes a distinction between data and constants:

- Constants can never be changed and must be provided when a model is defined. For example, a vector of known index values, such as for block indices, helps define the model graph itself and must be provided as constants. Variables used in the index ranges of for-loops must also be provided as constants.
- Data is a label for the role a node plays in the model. Nodes marked as data will by default be protected from any functions that would simulate over their values (see simulate in Chapter 13), but it is possible to over-ride that default or to change their values by direct assignment. This allows an algorithm to be applied to many data sets in the same model without re-creating the model each time. It also allows simulation of data in a model.

WinBUGS, OpenBUGS and JAGS do not allow data values to be changed or different nodes to be labeled as data without starting from the beginning again. Hence they do not distinguish between constants and data.

For compatibility with BUGS and JAGS, NIMBLE allows both to be provided in the constants argument to nimbleModel, in which case NIMBLE handles values for stochastic nodes as data and everything else as constants.

Values for nodes that appear only on the right-hand side of BUGS declarations (e.g., covariates/predictors) can be provided as constants or as data or initial values. There is no real difference between providing as data or initial values and the values can be added after building a model via setInits or setData.

6.1.1.2 Providing (or changing) data and initial values for an existing model

Whereas constants must be provided during the call to nimbleModel, data and initial values can be provided later via the model member functions setData and setInits. For example, if pumpData is a named list of data values (as above), then pump\$setData(pumpData) sets the named variables to the values in the list.

setData does two things: it sets the values of the data nodes, and it flags those nodes as containing data. nimbleFunction programmers can then use that information to control whether an algorithm should over-write data or not. For example, NIMBLE's simulate functions by default do not overwrite data values but can be told to do so. Values of data variables can be replaced, and the indication of which nodes should be treated as data can be reset by using the resetData method, e.g. pump\$resetData().

To change data values without any modification of which nodes are flagged as containing data, simply use R's usual assignment syntax to assign values in a compiled (or, more rarely, an uncompiled)

```
model, e.g.,
cModel$y <- c(1.5, 2.5, 1.7)
```

This can be useful for running an MCMC with a different dataset of the same size (and of course the same pattern of missingness, if any) without having to rebuild and recompile the MCMC, such as in a simulation study. It is possible to change the data values in a compiled model using setData, but we don't recommend doing this because setData won't modify which nodes are flagged as containing data in the already-constructed MCMC, thereby potentially introducing confusion.

6.1.1.3 Missing data values

Sometimes one needs a model variable to have a mix of data and non-data, often due to missing data values. In NIMBLE, when data values are provided, any nodes with NA values will *not* be labeled as data. A node following a multivariate distribution must be either entirely observed or entirely missing.

Here's an example of running an MCMC on the *pump* model, with two of the observations taken to be missing. Some of the steps in this example are documented more below. NIMBLE's default MCMC configuration will treat the missing values as unknowns to be sampled, as can be seen in the MCMC output here.

```
pumpMiss <- pump$newModel()</pre>
pumpMiss$resetData()
pumpDataNew <- pumpData</pre>
pumpDataNew$x[c(1, 3)] \leftarrow NA
pumpMiss$setData(pumpDataNew)
pumpMissConf <- configureMCMC(pumpMiss)</pre>
## ===== Monitors =====
## thin = 1: alpha, beta
## ===== Samplers =====
## RW sampler (1)
     - alpha
##
## posterior_predictive sampler (2)
     - theta[] (2 elements)
## conjugate sampler (9)
##
     - beta
##
     - theta[]
                (8 elements)
## ===== Comments =====
pumpMissConf$addMonitors('x', 'alpha', 'beta', 'theta')
## thin = 1: alpha, beta, theta, x
pumpMissMCMC <- buildMCMC(pumpMissConf)</pre>
Cobj <- compileNimble(pumpMiss, pumpMissMCMC)</pre>
niter <- 10
set.seed(0)
```

[4,]

[5,]

61

70

1

1

36

61

Cobj\$pumpMissMCMC\$run(niter) ## NULL samples <- as.matrix(Cobj\$pumpMissMCMC\$mvSamples)</pre> samples[1:5, 13:17] x[1] x[2] x[3] x[4] x[5]## ## [1,] 12 1 0 14 ## [2,] 3 7 1 35 14 ## [3,] 4 14 3 19 1

Missing values may also occur in explanatory/predictor variables. Values for such variables should be passed in via the data argument to nimbleModel, with NA for the missing values. In some contexts, one would want to specify distributions for such explanatory variables, for example so that an MCMC would impute the missing values.

6.1.1.4 Defining alternative models with the same code

14

14

3

3

Avoiding code duplication is a basic principle of good programming. In NIMBLE, one can use definition-time if-then-else statements to create different models from the same code. As a simple example, say we have a linear regression model and want to consider including or omitting x[2] as an explanatory variable:

```
regressionCode <- nimbleCode({</pre>
    intercept \sim dnorm(0, sd = 1000)
    slope1 \sim dnorm(0, sd = 1000)
    if(includeX2) {
        slope2 \sim dnorm(0, sd = 1000)
        for(i in 1:N)
             predictedY[i] <- intercept + slope1 * x1[i] + slope2 * x2[i]</pre>
    } else {
        for(i in 1:N) predictedY[i] <- intercept + slope1 * x1[i]</pre>
    sigmaY ~ dunif(0, 100)
    for(i in 1:N) Y[i] ~ dnorm(predictedY[i], sigmaY)
})
includeX2 <- FALSE
modelWithoutX2 <- nimbleModel(regressionCode, constants = list(N = 30),</pre>
                                check=FALSE)
modelWithoutX2$getVarNames()
## [1] "intercept"
                                             "slope1"
```

Whereas the constants are a property of the model definition – since they may help determine the model structure itself – data nodes can be different in different copies of the model generated from the same model definition. The setData and setInits described above can be used for each copy of the model.

6.1.1.5 Providing dimensions via nimbleModel

nimbleModel can usually determine the dimensions of every variable from the declarations in the BUGS code. However, it is possible to use a multivariate object only with empty indices (e.g. x[,]), in which case the dimensions must be provided as an argument to nimbleModel.

Here's an example with multivariate nodes. The first provides indices, so no dimensions argument is needed, while the second omits the indices and provides a dimensions argument instead.

```
code <- nimbleCode({</pre>
  y[1:K] \sim dmulti(p[1:K], n)
  p[1:K] ~ ddirch(alpha[1:K])
  log(alpha[1:K]) ~ dmnorm(alpha0[1:K], R[1:K, 1:K])
})
K <- 5
model <- nimbleModel(code, constants = list(n = 3, K = K,</pre>
                           alpha0 = rep(0, K), R = diag(K)),
                      check = FALSE)
codeAlt <- nimbleCode({</pre>
  y[] ~ dmulti(p[], n)
  p[] ~ ddirch(alpha[])
  log(alpha[]) ~ dmnorm(alpha0[], R[ , ])
})
model <- nimbleModel(codeAlt, constants = list(n = 3, K = K, alpha0 = rep(0, K),
                                                  R = diag(K)),
                                dimensions = list(y = K, p = K, alpha = K),
                                check = FALSE)
```

In that example, since alpha0 and R are provided as constants, we don't need to specify their dimensions.

6.1.2 Creating a model from standard BUGS and JAGS input files

Users with BUGS and JAGS experience may have files set up in standard formats for use in BUGS and JAGS. readBUGSmodel can read in the model, data/constant values and initial values in those formats. It can also take information directly from R objects somewhat more flexibly than nimbleModel, specifically allowing inputs set up similarly to those for BUGS and JAGS. In either case, after processing the inputs, it calls nimbleModel. Note that unlike BUGS and JAGS, only a single set of initial values can be specified in creating a model. Please see help(readBUGSmodel) for argument details.

As an example of using readBUGSmodel, let's create a model for the pump example from BUGS.

Note that readBUGSmodel allows one to include var and data blocks in the model file as in some of the BUGS examples (such as inhaler). The data block pre-computes constant and data values. Also note that if data and inits are provided as files, the files should contain R code that creates objects analogous to what would populate the list if a list were provided instead. Please see the JAGS manual examples or the classic_bugs directory in the NIMBLE package for example syntax. NIMBLE by and large does not need the information given in a var block but occasionally this is used to determine dimensionality, such as in the case of syntax like xbar <- mean(x[]) where x is a variable that appears only on the right-hand side of BUGS expressions.

Note that NIMBLE does not handle formatting such as in some of the original BUGS examples in which data was indicated with syntax such as data x in 'x.txt'.

6.1.3 Making multiple instances from the same model definition

Sometimes it is useful to have more than one copy of the same model. For example, an algorithm (i.e., nimbleFunction) such as an MCMC will be bound to a particular model before it is run. A user could build multiple algorithms to use the same model instance, or they may want each algorithm to have its own instance of the model.

There are two ways to create new instances of a model, shown in this example:

Each copy of the model can have different nodes flagged as data and different values in any nodes. They cannot have different values of N because that is a constant; it must be a constant because it helps define the model.

6.2 NIMBLE models are objects you can query and manipulate

NIMBLE models are objects that can be modified and manipulated from R. In this section we introduce some basic ways to use a model object. Chapter 13 covers more topics for writing algorithms that use models.

6.2.1 What are variables and nodes?

This section discusses some basic concepts and terminology to be able to speak about NIMBLE models clearly.

Suppose we have created a model from the following BUGS code.

```
mc <- nimbleCode({
    a ~ dnorm(0, 0.001)
    for(i in 1:5) {
        y[i] ~ dnorm(a, sd = 0.1)
        for(j in 1:3)
            z[i,j] ~ dnorm(y[i], tau)
    }
    tau ~ dunif(0, 20)
    y.squared[1:5] <- y[1:5]^2
})
model <- nimbleModel(mc, data = list(z = matrix(rnorm(15), nrow = 5)))</pre>
```

In NIMBLE terminology:

- The variables of this model are a, y, z, and y.squared.
- The nodes of this model are a, y[1],..., y[5], z[1,1],..., z[5, 3], and y.squared[1:5]. In graph terminology, nodes are vertices in the model graph.
- The node functions of this model are a ~ dnorm(0, 0.001), y[i] ~ dnorm(a, 0.1), z[i,j] ~ dnorm(y[i], sd = 0.1), and y.squared[1:5] <- y[1:5]^2. Each node's calculations are handled by a node function. Sometimes the distinction between nodes and node functions is important, but when it is not important we may refer to both simply as nodes.
- The *scalar elements* of this model include all the scalar nodes as well as the scalar elements y.squared[1],..., y.squared[5] of the multivariate node y.squared[1:5].

6.2.2 Determining the nodes and variables in a model

One can determine the variables in a model using getVarNames and the nodes in a model using getNodeNames. Optional arguments to getNodeNames allow you to select only certain types of nodes, as discussed in Section 13.1.1 and in the R help for getNodeNames.

```
model$getVarNames()
                                        "y"
## [1] "a"
## [3] "lifted_d1_over_sqrt_oPtau_cP" "z"
## [5] "tau"
                                        "y.squared"
model$getNodeNames()
    [1] "a"
                                         "tau"
##
                                         "y[2]"
    [3] "y[1]"
##
##
    [5] "y[3]"
                                         "y[4]"
    [7] "y[5]"
##
                                         "lifted_d1_over_sqrt_oPtau_cP"
   [9] "y.squared[1:5]"
                                         "z[1, 1]"
## [11] "z[1, 2]"
                                         "z[1, 3]"
                                         "z[2, 2]"
## [13] "z[2, 1]"
## [15] "z[2, 3]"
                                         "z[3, 1]"
## [17] "z[3, 2]"
                                         "z[3, 3]"
## [19] "z[4, 1]"
                                         "z[4, 2]"
## [21] "z[4, 3]"
                                         "z[5, 1]"
                                         "z[5, 3]"
## [23] "z[5, 2]"
```

Note that some of the nodes may be 'lifted' nodes introduced by NIMBLE (Section 13.1.2). In this case lifted d1_over_sqrt_oPtau_cP (this is a node for the standard deviation of the z nodes using NIMBLE's canonical parameterization of the normal distribution) is the only lifted node in the model.

To determine the dependencies of one or more nodes in the model, you can use getDependencies as discussed in Section 13.1.3.

6.2.3Accessing nodes

Model variables can be accessed and set just as in R using \$ and [[]]. For example

```
model$a <- 5
model$a
## [1] 5
model[["a"]]
## [1] 5
model\$y[2:4] \leftarrow rnorm(3)
model$y
                NA -0.7317482 0.8303732 -1.2080828
## [1]
                                                                NA
model[["y"]][c(1, 5)] \leftarrow rnorm(2)
model$y
## [1] -1.0479844 -0.7317482 0.8303732 -1.2080828 1.4411577
```

```
model$z[1,]
```

```
## [1] 2.0752450 0.2199248 -0.7660820
```

While nodes that are part of a variable can be accessed as above, each node also has its own name that can be used to access it directly. For example, y[2] has the name 'y[2]' and can be accessed by that name as follows:

```
model[["y[2]"]]
## [1] -0.7317482
model[["y[2]"]] <- -5
model$y

## [1] -1.0479844 -5.0000000  0.8303732 -1.2080828  1.4411577
model[["z[2, 3]"]]

## [1] -0.4302118
model[["z[2:4, 1:2]"]][1, 2]

## [1] -1.46725
model$z[2, 2]</pre>
```

[1] -1.46725

Notice that node names can include index blocks, such as model[["z[2:4, 1:2]"]], and these are not strictly required to correspond to actual nodes. Such blocks can be subsequently sub-indexed in the regular R manner, such as model[["z[2:4, 1:2]"]][1, 2].

6.2.4 How nodes are named

Every node has a name that is a character string including its indices, with a space after every comma. For example, X[1, 2, 3] has the name 'X[1, 2, 3]'. Nodes following multivariate distributions have names that include their index blocks. For example, a multivariate node for X[6:10, 3] has the name 'X[6:10, 3]'.

The definitive source for node names in a model is getNodeNames, described previously.

In the event you need to ensure that a name is formatted correctly, you can use the expandNodeNames method. For example, to get the spaces correctly inserted into X[1,1:5]:

```
## [1] "X[1, 1:5]"
```

Alternatively, for those inclined to R's less commonly used features, a nice trick is to use its parse and deparse functions.

```
deparse(parse(text = "X[1,1:5]", keep.source = FALSE)[[1]])
```

```
## [1] "X[1, 1:5]"
```

The keep.source = FALSE makes parse more efficient.

6.2.5 Why use node names?

Syntax like model[["z[2, 3]"]] may seem strange at first, because the natural habit of an R user would be model[["z"]][2,3]. To see its utility, consider the example of writing the nimbleFunction given in Section 2.8. By giving every scalar node a name, even if it is part of a multivariate variable, one can write functions in R or NIMBLE that access any single node by a name, regardless of the dimensionality of the variable in which it is embedded. This is particularly useful for NIMBLE, which resolves how to access a particular node during the compilation process.

6.2.6 Checking if a node holds data

Finally, you can query whether a node is flagged as data using the isData method applied to one or more nodes or nodes within variables:

Using models in parallel

[1] TRUE TRUE TRUE

6.3

NIMBLE uses Reference Classes and R6 classes for models and algorithms. Objects of these classes are passed by reference and copies of such objects are simply new variable names that reference the same underlying object.

Thus to run an algorithm in parallel on a given model, one must create multiple copies of the model and algorithm, and compiled versions of these, by calling nimbleModel, buildMCMC, compileNimble, etc. once for each copy. In other words all such calls should be within the parallelized block of code.

For a worked example in the context of MCMC, please see the parallelization example on our webpage.

Part III Algorithms in NIMBLE

Chapter 7

MCMC

NIMBLE provides a variety of paths to creating and executing an MCMC algorithm, which differ greatly in their simplicity of use, and also in the options available and customizability.

The most direct approach to invoking the MCMC engine is using the nimbleMCMC function (Section 7.1). This one-line call creates and executes an MCMC, and provides a wide range of options for controlling the MCMC: specifying monitors, burn-in, and thinning, running multiple MCMC chains with different initial values, and returning posterior samples, summary statistics, and/or a WAIC value. However, this approach is restricted to using NIMBLE's default MCMC algorithm; further customization of, for example, the specific samplers employed, is not possible.

The lengthier and more customizable approach to invoking the MCMC engine on a particular NIMBLE model object involves the following steps:

- 1. (Optional) Create and customize an MCMC configuration for a particular model:
 - a. Use configureMCMC to create an MCMC configuration (see Section 7.2). The configuration contains a list of samplers with the node(s) they will sample.
 - b. (Optional) Customize the MCMC configuration:
 - i. Add, remove, or re-order the list of samplers (Section 7.10 and help(samplers) in R for details), including adding your own samplers (Section 15.5);
 - ii. Change the tuning parameters or adaptive properties of individual samplers;
 - iii. Change the variables to monitor (record for output) and thinning intervals for MCMC samples.
- 2. Use buildMCMC to build the MCMC object and its samplers either from the model (using default MCMC configuration) or from a customized MCMC configuration (Section 7.3).
- 3. Compile the MCMC object (and the model), unless one is debugging and wishes to run the uncompiled MCMC.
- 4. Run the MCMC and extract the samples (Sections 7.4, 7.5 and 7.6).
- 5. Optionally, calculate the WAIC (Section 7.7).

Prior to version 0.8.0, NIMBLE provided two additional functions, MCMCsuite and compareMCMCs, to facilitate comparison of multiple MCMC algorithms, either internal or external to NIMBLE.

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Those capabilities have been redesigned and moved into a separate package called compareMCMCs.

End-to-end examples of MCMC in NIMBLE can be found in Sections 2.5-2.6 and Section 7.11.

7.1 One-line invocation of MCMC: nimbleMCMC

The most direct approach to executing an MCMC algorithm in NIMBLE is using nimbleMCMC. This single function can be used to create an underlying model and associated MCMC algorithm, compile both of these, execute the MCMC, and return samples, summary statistics, and a WAIC value. This approach circumvents the longer (and more flexible) approach using nimbleModel, configureMCMC, buildMCMC, compileNimble, and runMCMC, which is described subsequently.

The nimbleMCMC function provides control over the:

- number of MCMC iterations in each chain;
- number of MCMC chains to execute;
- number of burn-in samples to discard from each chain;
- thinning interval on which samples should be recorded;
- model variables to monitor and return posterior samples;
- initial values, or a function for generating initial values for each chain;
- setting the random number seed;
- returning posterior samples as a matrix or a coda mcmc object;
- returning posterior summary statistics; and
- returning a WAIC value calculated using post-burn-in samples from all chains.

This entry point for using nimbleMCMC is the code, constants, data, and inits arguments that are used for building a NIMBLE model (see Chapters 5 and 6). However, when using nimbleMCMC, the inits argument can also specify a list of lists of initial values that will be used for each MCMC chain, or a function that generates a list of initial values, which will be generated at the onset of each chain. As an alternative entry point, a NIMBLE model object can also be supplied to nimbleMCMC, in which case this model will be used to build the MCMC algorithm.

Based on its arguments, nimbleMCMC optionally returns any combination of

- Posterior samples,
- Posterior summary statistics, and
- · WAIC value.

The above are calculated and returned for each MCMC chain, using the post-burn-in and thinned samples. Additionally, posterior summary statistics are calculated for all chains combined when multiple chains are run.

Several example usages of nimbleMCMC are shown below:

```
code <- nimbleCode({
    mu ~ dnorm(0, sd = 1000)
    sigma ~ dunif(0, 1000)
    for(i in 1:10)
        x[i] ~ dnorm(mu, sd = sigma)
})
data <- list(x = c(2, 5, 3, 4, 1, 0, 1, 3, 5, 3))
initsFunction <- function() list(mu = rnorm(1,0,1), sigma = runif(1,0,10))</pre>
```

```
# execute one MCMC chain, monitoring the "mu" and "sigma" variables,
# with thinning interval 10. fix the random number seed for reproducible
# results. by default, only returns posterior samples.
mcmc.out <- nimbleMCMC(code = code, data = data, inits = initsFunction,</pre>
                       monitors = c("mu", "sigma"), thin = 10,
                       niter = 20000, nchains = 1, setSeed = TRUE)
# note that the inits argument to nimbleModel must be a list of
# initial values, whereas nimbleMCMC can accept inits as a function
# for generating new initial values for each chain.
initsList <- initsFunction()</pre>
Rmodel <- nimbleModel(code, data = data, inits = initsList)</pre>
# using the existing Rmodel object, execute three MCMC chains with
# specified burn-in. return samples, summary statistics, and WAIC.
mcmc.out <- nimbleMCMC(model = Rmodel,</pre>
                       niter = 20000, nchains = 3, nburnin = 2000,
                       summary = TRUE, WAIC = TRUE)
# run ten chains, generating random initial values for each
# chain using the inits function specified above.
# only return summary statistics from each chain; not all the samples.
mcmc.out <- nimbleMCMC(model = Rmodel, nchains = 10, inits = initsFunction,</pre>
                       samples = FALSE, summary = TRUE)
```

See help(nimbleMCMC) for further details.

7.2 The MCMC configuration

The MCMC configuration contains information needed for building an MCMC. When no customization is needed, one can jump directly to the buildMCMC step below. An MCMC configuration is an object of class MCMCconf, which includes:

- The model on which the MCMC will operate
- The model nodes which will be sampled (updated) by the MCMC
- The samplers and their internal configurations, called control parameters
- Two sets of variables that will be monitored (recorded) during execution of the MCMC and thinning intervals for how often each set will be recorded. Two sets are allowed because it can be useful to monitor different variables at different intervals

7.2.1 Default MCMC configuration

Assuming we have a model named Rmodel, the following will generate a default MCMC configuration:

```
mcmcConf <- configureMCMC(Rmodel)</pre>
```

The default configuration will contain a single sampler for each node in the model, and the default

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ordering follows the topological ordering of the model.

7.2.1.1 Default assignment of sampler algorithms

The default sampler assigned to a stochastic node is determined by the following, in order of precedence:

- 1. If the node has no data nodes in its entire downstream dependency network, a posterior_predictive sampler is assigned. This sampler updates the node in question and all downstream stochastic nodes by simulating new values from each node's conditional distribution. As of version 0.13.0, the operation of the posterior_predictive sampler has changed in order to improve MCMC mixing. See Section 7.2.1.2.
- 2. If the node has a conjugate relationship between its prior distribution and the distributions of its stochastic dependents, a conjugate ('Gibbs') sampler is assigned.
- 3. If the node follows a multinomial distribution, then a RW_multinomial sampler is assigned. This is a discrete random-walk sampler in the space of multinomial outcomes.
- 4. If a node follows a Dirichlet distribution, then a RW_dirichlet sampler is assigned. This is a random walk sampler in the space of the simplex defined by the Dirichlet.
- 5. If a node follows an LKJ correlation distribution, then a RW_block_lkj_corr_cholesky sampler is assigned. This is a block random walk sampler in a transformed space where the transformation uses the signed stickbreaking approach described in Section 10.12 of Stan Development Team (2021b).
- 6. If the node follows any other multivariate distribution, then a RW_block sampler is assigned for all elements. This is a Metropolis-Hastings adaptive random-walk sampler with a multivariate normal proposal (Roberts and Sahu, 1997).
- 7. If the node is binary-valued (strictly taking values 0 or 1), then a binary sampler is assigned. This sampler calculates the conditional probability for both possible node values and draws the new node value from the conditional distribution, in effect making a Gibbs sampler.
- 8. If the node is otherwise discrete-valued, then a slice sampler is assigned (Neal, 2003).
- 9. If none of the above criteria are satisfied, then a RW sampler is assigned. This is a Metropolis-Hastings adaptive random-walk sampler with a univariate normal proposal distribution.

Details of each sampler and its control parameters can be found by invoking help(samplers).

7.2.1.2 Sampling posterior predictive nodes

A posterior predictive node is a node that is not itself data and has no data nodes in its entire downstream (descendant) dependency network. Such nodes play no role in inference for model parameters but have often been included in BUGS models to accomplish posterior predictive checks and similar calculations.

As of version 0.13.0, NIMBLE's handling of posterior predictive nodes in MCMC sampling has changed in order to improve MCMC mixing. Samplers for nodes that are not posterior predictive nodes no longer condition on the values of the posterior predictive nodes. This produces a valid MCMC over the posterior distribution marginalizing over the posterior predictive nodes. This MCMC will generally mix better than an MCMC that conditions on the values of posterior predictive nodes, by reducing the dimensionality of the parameter space and removing the dependence between the sampled nodes and the posterior predictive nodes. At the end of each MCMC iteration, the posterior predictive nodes are sampled by posterior_predictive sampler(s) based on their conditional distribution(s).

7.2.1.3 Options to control default sampler assignments

Very basic control of default sampler assignments is provided via two arguments to configureMCMC. The useConjugacy argument controls whether conjugate samplers are assigned when possible, and the multivariateNodesAsScalars argument controls whether scalar elements of multivariate nodes are sampled individually. See help(configureMCMC) for usage details.

7.2.1.4 Default monitors

The default MCMC configuration includes monitors on all top-level stochastic nodes of the model. Only variables that are monitored will have their samples saved for use outside of the MCMC. MCMC configurations include two sets of monitors, each with different thinning intervals. By default, the second set of monitors (monitors2) is empty.

7.2.1.5 Automated parameter blocking

The automated parameter blocking algorithm is no longer actively maintained. In some cases, it may not operate correctly with more recent system features and/or distributions.

The default configuration may be replaced by one generated from an automated parameter blocking algorithm. This algorithm determines groupings of model nodes that, when jointly sampled with a RW_block sampler, increase overall MCMC efficiency. Overall efficiency is defined as the effective sample size of the slowest-mixing node divided by computation time. This is done by:

```
autoBlockConf <- configureMCMC(Rmodel, autoBlock = TRUE)</pre>
```

Note that this using autoBlock = TRUE compiles and runs MCMCs, progressively exploring different sampler assignments, so it takes some time and generates some output. It is most useful for determining effective blocking strategies that can be re-used for later runs. The additional control argument autoIt may also be provided to indicate the number of MCMC samples to be used in each trial of the automated blocking procedure (default 20,000).

7.2.2 Customizing the MCMC configuration

The MCMC configuration may be customized in a variety of ways, either through additional named arguments to configureMCMC or by calling methods of an existing MCMCconf object.

7.2.2.1 Controlling which nodes to sample

One can create an MCMC configuration with default samplers on just a particular set of nodes using the nodes argument to configureMCMC. The value for the nodes argument may be a character vector containing node and/or variable names. In the case of a variable name, a default sampler will be added for all stochastic nodes in the variable. The order of samplers will match the order of nodes. Any deterministic nodes will be ignored.

If a data node is included in nodes, it will be assigned a sampler. This is the only way in which a default sampler may be placed on a data node and will result in overwriting data values in the node.

7.2.2.2 Creating an empty configuration

If you plan to customize the choice of all samplers, it can be useful to obtain a configuration with no sampler assignments at all. This can be done by any of nodes = NULL, nodes = character(), or nodes = list().

7.2.2.3 Overriding the default sampler control list values

The default values of control list elements for all sampling algorithms may be overridden through use of the control argument to configureMCMC, which should be a named list. Named elements in the control argument will be used for all default samplers and any subsequent sampler added via addSampler (see below). For example, the following will create the default MCMC configuration, except all RW samplers will have their initial scale set to 3, and none of the samplers (RW, or otherwise) will be adaptive.

```
mcmcConf <- configureMCMC(Rmodel, control = list(scale = 3, adaptive = FALSE))</pre>
```

When adding samplers to a configuration using addSampler, the default control list can also be over-ridden.

7.2.2.4 Adding samplers to the configuration: addSampler

Additional samplers may be added to a configuration using the addSampler method of the MCMCconf object. The addSampler method has two modes of operation, determined by the default argument.

When default = TRUE, addSampler will assign NIMBLE's default sampling algorithm for each node specified following the same protocol as configureMCMC. This may include conjugate samplers, multivariate samplers, or otherwise, also using additional arguments to guide the selection process (for example, useConjugacy and multivariateNodesAsScalars). In this mode of operation, the type argument is not used.

When default = FALSE, or when this argument is omitted, addSampler uses the type argument to specify the precise sampler to assign. Instances of this particular sampler are assigned to all nodes specified. The type argument may be provided as a character string or a nimbleFunction object. Valid character strings are indicated in help(samplers) (do not include "sampler_"). Added samplers can be labeled with a name argument, which is used in output of printSamplers. Writing a new sampler as a nimbleFunction is covered in Section 15.5.

Regardless of the mode of operation, nodes are specified using either the target or the nodes argument. The target argument does not undergo expansion to constituent nodes (unless default = TRUE), and thus only a single sampler is added. The nodes argument is always expanded to the underlying nodes, and separate samplers are added for each node. Eithe argument is provided as a character vector. Newly added samplers will be appended to the end of current sampler list. Adding a sampler for a node will *not* remove existing samplers operating on that node.

The hierarchy of precedence for control list elements for added samplers is:

- 1. The control list argument provided to addSampler;
- 2. The original control list argument provided to configureMCMC;
- 3. The default values, as defined in the sampling algorithm setup function.

See help(addSampler) for more details.

7.2.2.5 Printing, re-ordering, modifying and removing samplers: printSamplers, re-moveSamplers, setSamplers, and getSamplerDefinition

The current, ordered, list of all samplers in the MCMC configuration may be printed by calling the printSamplers method. When you want to see only samplers acting on specific model nodes or variables, provide those names as an argument to printSamplers. The printSamplers method accepts arguments controlling the level of detail displayed as discussed in its R help information.

```
# Print all samplers
mcmcConf$printSamplers()

# Print all samplers operating on node "a[1]",
# or any of the "beta[]" variables
mcmcConf$printSamplers(c("a[1]", "beta"))

# Print all conjugate and slice samplers
mcmcConf$printSamplers(type = c("conjugate", "slice"))

# Print all RW samplers operating on "x"
mcmcConf$printSamplers("x", type = "RW")

# Print the first 100 samplers
mcmcConf$printSamplers(1:100)

# Print all samplers in their order of execution
mcmcConf$printSamplers(executionOrder = TRUE)
```

Samplers may be removed from the configuration object using removeSamplers, which accepts a character vector of node or variable names, or a numeric vector of indices.

```
# Remove all samplers acting on "x" or any component of it
mcmcConf$removeSamplers("x")

# Remove all samplers acting on "alpha[1]" and "beta[1]"
mcmcConf$removeSamplers(c("alpha[1]", "beta[1]"))

# Remove the first five samplers
mcmcConf$removeSamplers(1:5)

# Providing no argument removes all samplers
mcmcConf$removeSamplers()
```

Samplers to retain may be specified reordered using setSamplers, which also accepts a character vector of node or variable names, or a numeric vector of indices.

```
# Set the list of samplers to those acting on any components of the
# model variables "x", "y", or "z".
mcmcConf$setSamplers(c("x", "y", "z"))
# Set the list of samplers to only those acting on model nodes
```

```
# "alpha[1]", "alpha[2]", ..., "alpha[10]"
mcmcConf$setSamplers("alpha[1:10]")

# Truncate the current list of samplers to the first 10 and the 100th
mcmcConf$setSamplers(ind = c(1:10, 100))
```

The nimbleFunction definition underlying a particular sampler may be viewed using the getSamplerDefinition method, using the sampler index as an argument. A node name argument may also be supplied, in which case the definition of the first sampler acting on that node is returned. In all cases, getSamplerDefinition only returns the definition of the first sampler specified either by index or node name.

```
# Return the definition of the third sampler in the mcmcConf object
mcmcConf$getSamplerDefinition(3)

# Return the definition of the first sampler acting on node "x",
# or the first of any indexed nodes comprising the variable "x"
mcmcConf$getSamplerDefinition("x")
```

7.2.2.6 Customizing individual sampler configurations: getSamplers, setSamplers, setName, setSamplerFunction, setTarget, and setControl

Each sampler in an MCMCconf object is represented by a sampler configuration as a samplerConf object. Each samplerConf is a reference class object containing the following (required) fields: name (a character string), samplerFunction (a valid nimbleFunction sampler), target (the model node to be sampled), and control (list of control arguments). The MCMCconf method getSamplers allows access to the samplerConf objects. These can be modified and then passed as an argument to setSamplers to over-write the current list of samplers in the MCMC configuration object. However, no checking of the validity of this modified list is performed; if the list of samplerConf objects is corrupted to be invalid, incorrect behavior will result at the time of calling buildMCMC. The fields of a samplerConf object can be modified using the access functions setName(name), setSamplerFunction(fun), setTarget(target, model), and setControl(control).

Here are some examples:

```
# retrieve samplerConf list
samplerConfList <- mcmcConf$getSamplers()

# change the name of the first sampler
samplerConfList[[1]]$setName("newNameForThisSampler")

# change the sampler function of the second sampler,
# assuming existance of a nimbleFunction 'anotherSamplerNF',
# which represents a valid nimbleFunction sampler.
samplerConfList[[2]]$setSamplerFunction(anotherSamplerNF)

# change the 'adaptive' element of the control list of the third sampler
control <- samplerConfList[[3]]$control
control$adaptive <- FALSE</pre>
```

```
samplerConfList[[3]]$setControl(control)

# change the target node of the fourth sampler
samplerConfList[[4]]$setTarget("y", model) # model argument required

# use this modified list of samplerConf objects in the MCMC configuration
mcmcConf$setSamplers(samplerConfList)
```

7.2.2.7 Customizing the sampler execution order

The ordering of sampler execution can be controlled as well. This allows for sampler functions to execute multiple times within a single MCMC iteration, or the execution of different sampler functions to be interleaved with one another.

The sampler execution order is set using the function setSamplerExecutionOrder, and the current ordering of execution is retrieved using getSamplerExecutionOrder. For example, assuming the MCMC configuration object mcmcConf contains five samplers:

```
# first sampler to execute twice, in succession:
mcmcConf$setSamplerExecutionOrder(c(1, 1, 2, 3, 4, 5))

# first sampler to execute multiple times, interleaved:
mcmcConf$setSamplerExecutionOrder(c(1, 2, 1, 3, 1, 4, 1, 5))

# fourth sampler to execute 10 times, only
mcmcConf$setSamplerExecutionOrder(rep(4, 10))

# omitting the argument to setSamplerExecutionOrder()
# resets the ordering to each sampler executing once, sequentially
mcmcConf$setSamplerExecutionOrder()

# retrieve the current ordering of sampler execution
ordering <- mcmcConf$getSamplerExecutionOrder()

# print the sampler functions in the order of execution
mcmcConf$printSamplers(executionOrder = TRUE)</pre>
```

7.2.2.8 Monitors and thinning intervals: printMonitors, getMonitors, setMonitors, addMonitors, resetMonitors and setThin

An MCMC configuration object contains two independent sets of variables to monitor, each with their own thinning interval: thin corresponding to monitors, and thin2 corresponding to monitors2. Monitors operate at the variable level. Only entire model variables may be monitored. Specifying a monitor on a node, e.g., x[1], will result in the entire variable x being monitored.

The variables specified in monitors and monitors2 will be recorded (with thinning interval thin) into objects called mvSamples and mvSamples2, contained within the MCMC object. These are both modelValues objects; modelValues are NIMBLE data structures used to store multiple sets of

values of model variables¹. These can be accessed as the member data mvSamples and mvSamples2 of the MCMC object, and they can be converted to matrices using as.matrix or lists using as.list (see Section 7.6).

Monitors may be added to the MCMC configuration either in the original call to configureMCMC or using the addMonitors method:

A new set of monitor variables can be added to the MCMC configuration, overwriting the current monitors, using the setMonitors method:

```
# Replace old monitors, now monitor "delta" and "gamma" only
mcmcConf$setMonitors("gamma", "delta")
```

Similarly, either thinning interval may be set at either step:

```
# Using an argument to configureMCMC
mcmcConf <- configureMCMC(Rmodel, thin = 1, thin2 = 100)

# Calling a member method of the mcmcConf object
# This results in the same thinning intervals as above
mcmcConf$setThin(1)
mcmcConf$setThin2(100)</pre>
```

The current lists of monitors and thinning intervals may be displayed using the printMonitors method. Both sets of monitors (monitors and monitors2) may be reset to empty character vectors by calling the resetMonitors method. The methods getMonitors and getMonitors2 return the currently specified monitors and monitors2 as character vectors.

7.2.2.9 Monitoring model log-probabilities

To record model log-probabilities from an MCMC, one can add monitors for *logProb* variables (which begin with the prefix logProb_) that correspond to variables with (any) stochastic nodes. For example, to record and extract log-probabilities for the variables alpha, sigma_mu, and Y:

```
mcmcConf <- configureMCMC(Rmodel, enableWAIC = TRUE)
mcmcConf$addMonitors("logProb_alpha", "logProb_sigma_mu", "logProb_Y")
Rmcmc <- buildMCMC(mcmcConf)
Cmodel <- compileNimble(Rmodel)
Cmcmc <- compileNimble(Rmcmc, project = Rmodel)
Cmcmc$run(10000)
samples <- as.matrix(Cmcmc$mvSamples)</pre>
```

¹See Section 14.1 for general information on modelValues.

The samples matrix will contain both MCMC samples and model log-probabilities.

7.3 Building and compiling the MCMC

Once the MCMC configuration object has been created, and customized to one's liking, it may be used to build an MCMC function:

```
Rmcmc <- buildMCMC(mcmcConf)</pre>
```

buildMCMC is a nimbleFunction. The returned object Rmcmc is an instance of the nimbleFunction specific to configuration mcmcConf (and of course its associated model).

Note that if you would like to be able to calculate the WAIC of the model, you should usually set enableWAIC = TRUE as an argument toconfigureMCMC (or to buildMCMC if not using configureMCMC), or set nimbleOptions(MCMCenableWAIC = TRUE), which will enable WAIC calculations for all subsequently built MCMC functions. For more information on WAIC calculations, including situations in which you can calculate WAIC without having set enableWAIC = TRUE see Section 7.7 or help(waic) in R.

When no customization is needed, one can skip configureMCMC and simply provide a model object to buildMCMC. The following two MCMC functions will be identical:

```
mcmcConf <- configureMCMC(Rmodel) # default MCMC configuration
Rmcmc1 <- buildMCMC(mcmcConf)

Rmcmc2 <- buildMCMC(Rmodel) # uses the default configuration for Rmodel</pre>
```

For speed of execution, we usually want to compile the MCMC function to C++ (as is the case for other NIMBLE functions). To do so, we use compileNimble. If the model has already been compiled, it should be provided as the project argument so the MCMC will be part of the same compiled project. A typical compilation call looks like:

```
Cmcmc <- compileNimble(Rmcmc, project = Rmodel)</pre>
```

Alternatively, if the model has not already been compiled, they can be compiled together in one line:

```
Cmcmc <- compileNimble(Rmodel, Rmcmc)</pre>
```

Note that if you compile the MCMC with another object (the model in this case), you'll need to explicitly refer to the MCMC component of the resulting object to be able to run the MCMC:

```
Cmcmc$Rmcmc$run(niter = 1000)
```

7.4 User-friendly execution of MCMC algorithms: runMCMC

Once an MCMC algorithm has been created using buildMCMC, the function runMCMC can be used to run multiple chains and extract posterior samples, summary statistics and/or a WAIC value. This is a simpler approach to executing an MCMC algorithm, than the process of executing and extracting samples as described in Sections 7.5 and 7.6.

runMCMC also provides several user-friendly options such as burn-in, thinning, running multiple chains, and different initial values for each chain. However, using runMCMC does not support several lower-level options, such as timing the individual samplers internal to the MCMC, continuing an existing MCMC run (picking up where it left off), or modifying the sampler execution ordering.

runMCMC takes arguments that will control the following aspects of the MCMC:

- Number of iterations in each chain;
- Number of chains;
- Number of burn-in samples to discard from each chain;
- Thinning interval for recording samples;
- Initial values, or a function for generating initial values for each chain;
- Setting the random number seed;
- Returning the posterior samples as a coda mcmc object;
- Returning summary statistics calculated from each chains; and
- Returning a WAIC value calculated using (post-burn-in) samples from all chains.

The following examples demonstrate some uses of runMCMC, and assume the existence of Cmcmc, a compiled MCMC algorithm.

```
# run a single chain, and return a matrix of samples
mcmc.out <- runMCMC(Cmcmc)</pre>
# run three chains of 10000 samples, discard initial burn-in of 1000,
# record samples thereafter using a thinning interval of 10,
# and return of list of sample matrices
mcmc.out <- runMCMC(Cmcmc, niter=10000, nburnin=1000, thin=10, nchains=3)</pre>
# run three chains, returning posterior samples, summary statistics,
# and the WAIC value
mcmc.out <- runMCMC(Cmcmc, nchains = 3, summary = TRUE, WAIC = TRUE)</pre>
# run two chains, and specify the initial values for each
initsList <- list(list(mu = 1, sigma = 1),</pre>
                   list(mu = 2, sigma = 10))
mcmc.out <- runMCMC(Cmcmc, nchains = 2, inits = initsList)</pre>
# run ten chains of 100,000 iterations each, using a function to
# generate initial values and a fixed random number seed for each chain.
# only return summary statistics from each chain; not all the samples.
initsFunction <- function()</pre>
    list(mu = rnorm(1,0,1), sigma = runif(1,0,100))
mcmc.out <- runMCMC(Cmcmc, niter = 100000, nchains = 10,</pre>
                     inits = initsFunction, setSeed = TRUE,
                     samples = FALSE, summary = TRUE)
```

See help(runMCMC) for further details.

7.5 Running the MCMC

The MCMC algorithm (either the compiled or uncompiled version) can be executed using the member method mcmc\$run (see help(buildMCMC) in R). The run method has one required argument, niter, the number of iterations to be run.

The run method has optional arguments nburnin, thin and thin2. These can be used to specify the number of pre-thinning burn-in samples to discard, and the post-burnin thinning intervals for recording samples (corresponding to monitors and monitors2). If either thin and thin2 are provided, they will override the thinning intervals that were specified in the original MCMC configuration object.

7.5.1 Rerunning versus restarting an MCMC

The run method has an optional reset argument. When reset = TRUE (the default value), the following occurs prior to running the MCMC:

- 1. All model nodes are checked and filled or updated as needed, in valid (topological) order. If a stochastic node is missing a value, it is populated using a call to simulate and its log probability value is calculated. The values of deterministic nodes are calculated from their parent nodes. If any right-hand-side-only nodes (e.g., explanatory variables) are missing a value, an error results.
- 2. All MCMC sampler functions are reset to their initial state: the initial values of any sampler control parameters (e.g., scale, sliceWidth, or propCov) are reset to their initial values, as were specified by the original MCMC configuration.
- The internal modelValues objects mvSamples and mvSamples2 are each resized to the appropriate length for holding the requested number of samples (niter/thin, and niter/thin2, respectively).

This means that one can begin a new run of an existing MCMC without having to rebuild or recompile the model or the MCMC. This can be helpful if one wants to use the same model and MCMC configuration, but with different initial values, different values of data nodes (but which nodes are data nodes must be the same), changes to covariate values(or other non-data, non-parameter values) in the model, or a different number of MCMC iterations, thinning interval or burn-in.

In contrast, when mcmc\$run(niter, reset = FALSE) is called, the MCMC picks up from where it left off, continuing the previous chain and expanding the output as needed. No values in the model are checked or altered, and sampler functions are not reset to their initial states.

The run method also has an optional resetMV argument. This argument is only considered when 'reset is set to FALSE. When mcmc\$run(niter, reset = FALSE, resetMV = TRUE) is called the internal modelValues objects mvSamples and mvSamples2 are each resized to the appropriate length for holding the requested number of samples (niter/thin, and niter/thin2, respectively) and the MCMC carries on from where it left off. In other words, the previously obtained samples are deleted (e.g. to reduce memory usage) prior to continuing the MCMC. The default value of resetMV is FALSE.

7.5.2 Measuring sampler computation times: getTimes

If you want to obtain the computation time spent in each sampler, you can set time=TRUE as a run-time argument and then use the method getTimes() obtain the times. For example,

```
Cmcmc$run(niter, time = TRUE)
Cmcmc$getTimes()
```

will return a vector of the total time spent in each sampler, measured in seconds.

7.5.3 Assessing the adaption process of RW and RW_block samplers

If you'd like to see the evolution (over the iterations) of the acceptance proportion and proposal scale information, you can use some internal methods provided by NIMBLE, after setting two options to make the history accessible. Here we assume that cMCMC is the compiled MCMC object and idx is the numeric index of the sampler function you want to access from amongst the list of sampler functions that are part of the MCMC.

```
## set options to make history accessible
nimbleOptions(buildInterfacesForCompiledNestedNimbleFunctions = TRUE)
nimbleOptions(MCMCsaveHistory = TRUE)
## Next, set up and run your MCMC.
## Now access the history information:
Cmcmc$samplerFunctions[[idx]]$getScaleHistory()
Cmcmc$samplerFunctions[[idx]]$getAcceptanceHistory()
Cmcmc$samplerFunctions[[idx]]$getPropCovHistory() ## only for RW_block
```

Note that modifying elements of the control list may greatly affect the performance of the RW_block sampler. In particular, the sampler can take a long time to find a good proposal covariance when the elements being sampled are not on the same scale. We recommend providing an informed value for 'propCov' in this case (possibly simply a diagonal matrix that approximates the relative scales), as well as possibly providing a value of 'scale' that errs on the side of being too small. You may also consider decreasing 'adaptFactorExponent' and/or 'adaptInterval', as doing so has greatly improved performance in some cases.

7.6 Extracting MCMC samples

After executing the MCMC, the output samples can be extracted as follows:

```
mvSamples <- mcmc$mvSamples
mvSamples2 <- mcmc$mvSamples2</pre>
```

These model Values objects can be converted into matrices using as.matrix or lists using as.list:

```
samplesMatrix <- as.matrix(mvSamples)
samplesList <- as.list(mvSamples)
samplesMatrix2 <- as.matrix(mvSamples2)
samplesList2 <- as.list(mvSamples2)</pre>
```

The column names of the matrices will be the node names of nodes in the monitored variables. Then, for example, the mean of the samples for node x[2] could be calculated as:

mean(samplesMatrix[, "x[2]"])

The list version will contain an element for each variable that will be the size and shape of the variable with an additional index for MCMC iteration. By default MCMC iteration will be the first index, but including iterationAsLastIndex = TRUE will make it the last index.

Obtaining samples as matrices or lists is most common, but see Section 14.1 for more about programming with modelValues objects, especially if you want to write nimbleFunctions to use the samples.

7.7 Calculating WAIC

The WAIC (Watanabe, 2010) can be calculated from the posterior samples produced during the MCMC algorithm. Users have two options to calculate WAIC. The main approach requires enabling WAIC when setting up the MCMC and allows access to a variety of ways to calculate WAIC. This approach does not require that any specific monitors be set² because the WAIC is calculated in an online manner, accumulating the necessary quantities to calculate WAIC as the MCMC is run.

The second approach allows one to calculate the WAIC after an MCMC has been run using an MCMC object or matrix (or dataframe) containing the posterior samples, but it requires the user to have correctly specified the monitored variables and only provides the default way to calculate WAIC. An advantage of the second approach is that one can specify additional burnin beyond that specified for the MCMC.

Here we first discuss the main approach and then close the section by showing the second approach. Specific details of the syntax are provided in help(waic).

To enable WAIC for the first approach, the argument enableWAIC = TRUE must be supplied to configureMCMC (or to buildMCMC if not using configureWAIC), or the MCMCenableWAIC NIMBLE option must have been set to TRUE.

The WAIC (as well as the pWAIC and lppd values) is extracted by the member method mcmc\$getWAIC (see help(waic) in R for more details) or is available as the WAIC element of the runMCMC or nimbleMCMC output lists. One can use the member method mcmc\$getWAICdetails for additional quantities related to WAIC that are discussed below.

Note that there is not a unique value of WAIC for a model. WAIC is calculated from Equations 5, 12, and 13 in Gelman et al. (2014) (i.e., using pWAIC2), as discussed in detail in Hug and Paciorek (2021). Therefore, NIMBLE provides user control over how WAIC is calculated in two ways.

First, by default NIMBLE provides the conditional WAIC, namely the version of WAIC where all parameters directly involved in the likelihood are treated as θ for the purposes of Equation 5 from Gelman et al. (2014). Users can request the marginal WAIC (see Ariyo et al. (2020)), namely the version of WAIC where latent variables are integrated over (i.e., using a marginal likelihood). This is done by providing the waicControl list with a marginalizeNodes element to configureMCMC or buildMCMC (when providing a model as the argument to buildMCMC). See help(waic) for more details.

 $^{^{2}}$ Prior to version 0.12.0, NIMBLE required specific monitors, because WAIC was calculated at the end of the MCMC using the entire set of MCMC samples.

Second, WAIC relies on a partition of the observations, i.e., 'pointwise' prediction. By default, in NIMBLE the sum over log pointwise predictive density values treats each data node as contributing a single value to the sum. When a data node is multivariate, that data node contributes a single value to the sum based on the joint density of the elements in the node. If one wants to group data nodes such that the joint density within each group is used, one can provide the waicControl list with a dataGroups element to configureMCMC or buildMCMC (when providing a model as the argument to buildMCMC). See help(waic) for more details.

Note that based on a limited set of simulation experiments in Hug and Paciorek (2021), our tentative recommendation is that users only use marginal WAIC if also using grouping.

Marginal WAIC requires using Monte Carlo simulation at each iteration of the MCMC to average over draws for the latent variables. To assess the stability of the marginal WAIC to the number of Monte Carlo iterations, one can examine how the WAIC changes with increasing iterations (up to the full number of iterations specified via the nltsMarginal element of the waicControl list) based on the WAIC_partialMC, lppd_partialMC, and pWAIC_partialMC elements of the detailed WAIC output.

For comparing WAIC between two models, Vehtari et al. (2017) discuss using the per-observation (more generally, per-data group) contributions to the overall WAIC values to get an approximate standard error for the difference in WAIC between the models. These element-wise values are available as the WAIC_elements, lppd_elements, and pWAIC_elements components of the detailed WAIC output.

The second overall approach to WAIC available in NIMBLE allows one to calculate WAIC post hoc after MCMC sampling using an MCMC object or matrix (or dataframe) containing posterior samples. Simply set up the MCMC and run it without enabling WAIC (but making sure to include in the monitors all stochastic parent nodes of the data nodes) and then use the function calculateWAIC, as shown in this example:

```
samples <- runMCMC(Cmcmc, niter = 10000, nburnin = 1000)
## Using calculateWAIC with an MCMC object
calculateWAIC(Cmcmc)
## Using calculateWAIC with a matrix of samples
calculateWAIC(samples, model)
## Specifying additional burnin, so only last 5000 (1000+4000) iterations used
calculateWAIC(Cmcmc, burnin = 4000)</pre>
```

This approach only provides conditional WAIC without any grouping of data nodes (though one can achieve grouping by grouping data nodes into multivariate nodes).

7.8 k-fold cross-validation

The runCrossValidate function in NIMBLE performs k-fold cross-validation on a nimbleModel fit via MCMC. More information can be found by calling help(runCrossValidate).

7.9 Variable selection using Reversible Jump MCMC

A common method for Bayesian variable selection in regression-style problems is to define a space of different models and have MCMC sample over the models as well as the parameters for each model.

The models differ in which explanatory variables are included. Often this idea is implemented in the BUGS language by writing the largest possible model and including indicator variables to turn regression parameters off and on (see model code below for an example of how indicator variables can be used). Foramally, that approach doesn't sample between different models, instead embedding the models in one large model. However, that approach can result in poor mixing and require compromises in choices of prior distributions because a given regression parameter will sample from its prior when the corresponding indicator is 0. It is also computationally wasteful, because sampling effort will be spent on a coefficient even if it has no effect because the corresponding indicator is 0.

A different view of the problem is that the different combinations of coefficients represent models of different dimensions. Reversible Jump MCMC (Green, 1995) (RJMCMC) is a general framework for MCMC simulation in which the dimension of the parameter space can vary between iterates of the Markov chain. The reversible jump sampler can be viewed as an extension of the Metropolis-Hastings algorithm onto more general state spaces. NIMBLE provides an implementation of RJMCMC for variable selection that requires the user to write the largest model of interest and supports use of indicator variables but formally uses RJMCMC sampling for better mixing and efficiency. When a coefficient is not in the model (or its indicator is 0), it will not be sampled, and it will therefore not follow its prior in that case.

In technical detail, given two models M_1 and M_2 of possibly different dimensions, the core idea of RJMCMC is to remove the difference in the dimensions of models M_1 and M_2 by supplementing the corresponding parameters θ_1 and θ_2 with auxiliary variables $u_{1\to 2}$ and $u_{2\to 1}$ such that $(\theta_1, u_{1\to 2})$ and $(\theta_2, u_{2\to 1})$ are in bijection, $(\theta_2, u_{2\to 1}) = \Psi(\theta_1, u_{1\to 2})$. The corresponding Metropolis-Hastings acceptance probability is generalized accounting for the proposal density for the auxiliary variables.

NIMBLE implements RJMCMC for variable selection using a univariate normal distribution centered on 0 (or some fixed value) as the proposal density for parameters being added to the model. Two ways to set up models for RJMCMC are supported, which differ by whether the inclusion probabilities for each parameter are assumed known or must be written in the model:

- If the inclusion probabilities are assumed known, then RJMCMC may be used with regular model code, i.e. model code written without heed to variable selection.
- If the inclusion probability is a parameter in the model, perhaps with its own prior, then RJMCMC requires that indicator variables be written in the model code. The indicator variables will be sampled using RJMCMC, but otherwise they are like any other nodes in the model.

The steps to set up variable selection using RJMCMC are:

- 1. Write the model with indicator variables if needed.
- 2. Configure the MCMC as usual with configureMCMC.
- 3. Modify the resulting MCMC configuration object with configureRJ.

The configureRJ function modifies the MCMC configuration to (1) assign samplers that turn on and off variable in the model and (2) modify the existing samplers for the regression coefficients to use 'toggled' versions. The toggled versions invoke the samplers only when corresponding variable is currently in the model. In the case where indicator variables are included in the model, sampling to turn on and off variables is done using RJ_indicator samplers. In the case where indicator variables are not included, sampling is done using RJ fixed prior samplers.

In the following we provide an example for the two different model specifications.

7.9.1 Using indicator variables

Here we consider a normal linear regression in which two covariates x1 and x2 are candidates to be included in the model. The two corresponding coefficients are beta1 and beta2. The indicator variables are z1 and z2, and their inclusion probability is psi. As described below, one can also use vectors for a set of coefficients and corresponding indicator variables.

```
## Linear regression with intercept and two covariates
code <- nimbleCode({</pre>
 beta0 ~ dnorm(0, sd = 100)
 beta1 ~ dnorm(0, sd = 100)
 beta2 \sim dnorm(0, sd = 100)
  sigma \sim dunif(0, 100)
  z1 ~ dbern(psi) ## indicator variable associated with beta1
 z2 ~ dbern(psi) ## indicator variable associated with beta2
 psi ~ dbeta(1, 1) ## hyperprior on inclusion probability
  for(i in 1:N) {
    Ypred[i] <- beta0 + beta1 * z1 * x1[i] + beta2 * z2 * x2[i]</pre>
    Y[i] ~ dnorm(Ypred[i], sd = sigma)
})
## simulate some data
set.seed(1)
N <- 100
x1 \leftarrow runif(N, -1, 1)
x2 <- runif(N, -1, 1) ## this covariate is not included
Y \leftarrow rnorm(N, 1.5 + 2 * x1, sd = 1)
## build the model and configure default MCMC
RJexampleModel <- nimbleModel(code, constants = list(N = N),</pre>
                               data = list(Y = Y, x1 = x1, x2 = x2),
                               inits = list(beta0 = 0, beta1 = 0, beta2 = 0,
                               sigma = sd(Y), z2 = 1, z1 = 1, psi = 0.5)
RJexampleConf <- configureMCMC(RJexampleModel)</pre>
## For illustration, look at the default sampler assignments
RJexampleConf$printSamplers()
## [1] conjugate_dnorm_dnorm_additive sampler: beta0
## [2] conjugate_dnorm_dnorm_linear sampler: beta1
## [3] conjugate_dnorm_dnorm_linear sampler: beta2
## [4] RW sampler: sigma
## [5] conjugate_dbeta_dbern_identity sampler: psi
## [6] binary sampler: z1
## [7] binary sampler: z2
```

At this point we can modify the MCMC configuration object, RJexampleConf, to use reversible jump samplers for selection on beta1 and beta2.

The indicatorNodes gives the corresponding indicator nodes, ordered to match targetNodes. The control list gives the means and scale (standard deviation) for normal reversible jump proposals for targetNodes. The means will typically be 0 (by default mean = 0 and scale = 1), but they could be anything. An optional control element called fixedValue can be provided in the non-indicator setting; this gives the value taken by nodes in targetNodes when they are out of the model (by default, fixedValue is 0). All control elements can be scalars or vectors. A scalar values will be used for all targetNodes. A vector value must be of equal length as targetNodes and will be used in order.

To use RJMCMC on a vector of coefficients with a corresponding vector of indicator variables, simply provide the variable names for targetNodes and indicatorNodes. For example, targetNodes = "beta" is equivalent to targetNodes = c("beta[1]", "beta[2]") and indicatorNodes = "z" is equivalent to indicatorNodes = c("z[1]", "z[2]"). Expansion of variable names into a vector of node names will occur as described in Section 13.3.1.1. When using this method, both arguments must be provided as variable names to be expanded.

Next we can see the result of configureRJ by printing the modified list of samplers:

RJexampleConf\$printSamplers()

```
## [1] conjugate_dnorm_dnorm_additive sampler: beta0
## [2] RW sampler: sigma
## [3] conjugate_dbeta_dbern_identity sampler: psi
## [4] RJ_indicator sampler: z1, mean: 0, scale: 2, targetNode: beta1
## [5] RJ_toggled sampler: beta1, samplerType: conjugate_dnorm_dnorm_linear
## [6] RJ_indicator sampler: z2, mean: 0, scale: 2, targetNode: beta2
## [7] RJ_toggled sampler: beta2, samplerType: conjugate_dnorm_dnorm_linear
```

An RJ_indicator sampler was assigned to each of z[1] and z[2] in place of the binary sampler, while the samplers for beta[1] and beta[2] have been changed to RJ_toggled samplers. The latter samplers contain the original samplers, in this case conjugate_dnorm_dnorm samplers, and use them only when the corresponding indicator variable is equal to 1 (i.e., when the coefficient is in the model).

Notice that the order of the samplers has changed, since configureRJ calls removeSampler for nodes in targetNodes and indicatorNodes, and subsequently addSampler, which appends the sampler to the end of current sampler list. Order can be modified by using setSamplers.

Also note that configureRJ modifies the MCMC configuration (first argument) and returns NULL.

7.9.2 Without indicator variables

We consider the same regression setting, but without the use of indicator variables and with fixed probabilities of including each coefficient in the model.

```
## Linear regression with intercept and two covariates
code <- nimbleCode({</pre>
  beta0 ~ dnorm(0, sd = 100)
 beta1 ~ dnorm(0, sd = 100)
 beta2 \sim dnorm(0, sd = 100)
  sigma ~ dunif(0, 100)
 for(i in 1:N) {
    Ypred[i] <- beta0 + beta1 * x1[i] + beta2 * x2[i]</pre>
    Y[i] ~ dnorm(Ypred[i], sd = sigma)
  }
})
## build the model
RJexampleModel2 <- nimbleModel(code, constants = list(N = N),</pre>
                                 data = list(Y = Y, x1 = x1, x2 = x2),
                                 inits = list(beta0 = 0, beta1 = 0, beta2 = 0,
                                               sigma = sd(Y))
RJexampleConf2 <- configureMCMC(RJexampleModel2)</pre>
## print NIMBLE default samplers
RJexampleConf2$printSamplers()
## [1] conjugate_dnorm_dnorm_additive sampler: beta0
## [2] conjugate_dnorm_dnorm_linear sampler: beta1
## [3] conjugate_dnorm_dnorm_linear sampler: beta2
## [4] RW sampler: sigma
In this case, since there are no indicator variables, we need to pass to configureRJ the prior
inclusion probabilities for each node in targetNodes, by specifying either one common value or a
vector of values for the argument priorProb. This case does not allow for a stochastic prior.
configureRJ(conf = RJexampleConf2,
            targetNodes = c("beta1", "beta2"),
```

```
## [1] conjugate_dnorm_dnorm_additive sampler: beta0
## [2] RW sampler: sigma
## [3] RJ_fixed_prior sampler: beta1, priorProb: 0.5, mean: 0, scale: 2, fixedValue: 1.5
## [4] RJ_toggled sampler: beta1, samplerType: conjugate_dnorm_dnorm_linear, fixedValue: 1.5
## [5] RJ_fixed_prior sampler: beta2, priorProb: 0.5, mean: 0, scale: 2, fixedValue: 0
## [6] RJ_toggled sampler: beta2, samplerType: conjugate_dnorm_dnorm_linear, fixedValue: 0
```

Since there are no indicator variables, the RJ_fixed_prior sampler is assigned directly to each of beta[1] and beta[2] along with the RJ_toggled sampler. The former sets a coefficient to its fixedValue when it is out of the model. The latter invokes the regular sampler for the coefficient only if it is in the model at a given iteration.

If fixedValue is given when using indicatorNodes the values provided in fixedValue are ignored. However the same behavior can be obtained in this situation, using a different model specification. For example, the model in 7.9.1 can be modified to have beta1 equal to 1.5 when not in the model as follows:

7.10 Samplers provided with NIMBLE

Most documentation of MCMC samplers provided with NIMBLE can be found by invoking help(samplers) in R. Here we provide additional explanation of conjugate samplers and how complete customization can be achieved by making a sampler use an arbitrary log-likelihood function, such as to build a particle MCMC algorithm.

7.10.1 Conjugate ('Gibbs') samplers

By default, configureMCMC() and buildMCMC() will assign conjugate samplers to all nodes satisfying a conjugate relationship, unless the option useConjugacy = FALSE is specified.

The current release of NIMBLE supports conjugate sampling of the relationships listed in Table 7.1^3 .

Table 7.1: Conjugate	relationships	supported	by NIMBLE's
MCMC engine.			

Prior Distribution	Sampling (Dependent Node) Distribution	Parameter
Beta	Bernoulli	prob
	Binomial	prob
	Negative Binomial	prob
Dirichlet	Multinomial	prob
	Categorical	prob
Flat	Normal	mean
	Lognormal	meanlog
Gamma	Poisson	lambda
	Normal	tau
	Lognormal	taulog
	Gamma	rate
	Inverse Gamma	scale

³NIMBLE's internal definitions of these relationships can be viewed with nimble:::conjugacyRelationshipsInputList.

Prior Distribution	Sampling (Dependent Node) Distribution	Parameter
	Exponential	rate
	Double Exponential	rate
	Weibull	lambda
Halfflat	Normal	sd
	Lognormal	sdlog
Inverse Gamma	Normal	var
	Lognormal	varlog
	Gamma	scale
	Inverse Gamma	rate
	Exponential	scale
	Double Exponential	scale
Normal	Normal	mean
	Lognormal	meanlog
Multivariate Normal	Multivariate Normal	mean
Wishart	Multivariate Normal	prec
Inverse Wishart	Multivariate Normal	cov

Conjugate sampler functions may (optionally) dynamically check that their own posterior likelihood calculations are correct. If incorrect, a warning is issued. However, this functionality will roughly double the run-time required for conjugate sampling. By default, this option is disabled in NIMBLE. This option may be enabled with nimbleOptions(verifyConjugatePosteriors = TRUE).

If one wants information about conjugate node relationships for other purposes, they can be obtained using the <code>checkConjugacy</code> method on a model. This returns a named list describing all conjugate relationships. The <code>checkConjugacy</code> method can also accept a character vector argument specifying a subset of node names to check for conjugacy.

7.10.2 Customized log-likelihood evaluations: RW_llFunction sampler

Sometimes it is useful to control the log-likelihood calculations used for an MCMC updater instead of simply using the model. For example, one could use a sampler with a log-likelihood that analytically (or numerically) integrates over latent model nodes. Or one could use a sampler with a log-likelihood that comes from a stochastic approximation such as a particle filter (see below), allowing composition of a particle MCMC (PMCMC) algorithm (Andrieu et al., 2010). The RW_11Function sampler handles this by using a Metropolis-Hastings algorithm with a normal proposal distribution and a user-provided log-likelihood function. To allow compiled execution, the log-likelihood function must be provided as a specialized instance of a nimbleFunction. The log-likelihood function may use the same model as the MCMC as a setup argument (as does the example below), but if so the state of the model should be unchanged during execution of the function (or you must understand the implications otherwise).

The RW_llFunction sampler can be customized using the control list argument to set the initial proposal distribution scale and the adaptive properties for the Metropolis-Hastings sampling. In addition, the control list argument must contain a named llFunction element. This is the specialized nimbleFunction that calculates the log-likelihood; it must accept no arguments and return a scalar double number. The return value must be the total log-likelihood of all stochastic dependents of the target nodes – and, if includesTarget = TRUE, of the target node(s) themselves

- or whatever surrogate is being used for the total log-likelihood. This is a required control list element with no default. See help(samplers) for details.

Here is a complete example:

```
code <- nimbleCode({</pre>
    p \sim dunif(0, 1)
    y ~ dbin(p, n)
})
Rmodel <- nimbleModel(code, data = list(y=3), inits = list(p=0.5, n=10))
llFun <- nimbleFunction(</pre>
    setup = function(model) { },
    run = function() {
        y <- model$y
        p <- model$p
        n <- model$n
        11 <- lfactorial(n) - lfactorial(y) - lfactorial(n-y) +</pre>
               y * log(p) + (n-y) * log(1-p)
        returnType(double())
        return(ll[1])
    }
RllFun <- llFun(Rmodel)
mcmcConf <- configureMCMC(Rmodel, nodes = NULL)</pre>
mcmcConf$addSampler(target = "p", type = "RW_llFunction",
    control = list(llFunction = RllFun, includesTarget = FALSE))
Rmcmc <- buildMCMC(mcmcConf)</pre>
```

Note that we need to return 11[1] and not just 11 because there are no scalar variables in compiled models. Hence y and other variables, and therefore 11, are of dimension 1 (i.e., vectors / one-dimensional arrays), so we need to specify the first element in order to have the return type be a scalar.

7.10.3 Particle MCMC sampler

For state space models, a particle MCMC (PMCMC) sampler can be specified for top-level parameters. This sampler is described in Section 8.1.2.

7.11 Detailed MCMC example: *litters*

Here is a detailed example of specifying, building, compiling, and running two MCMC algorithms. We use the litters example from the BUGS examples.

```
################################
##### model configuration #####
################################
# define our model using BUGS syntax
litters_code <- nimbleCode({</pre>
   for (i in 1:G) {
       a[i] ~ dgamma(1, .001)
       b[i] ~ dgamma(1, .001)
       for (j in 1:N) {
           r[i,j] ~ dbin(p[i,j], n[i,j])
           p[i,j] ~ dbeta(a[i], b[i])
       mu[i] \leftarrow a[i] / (a[i] + b[i])
       theta[i] \leftarrow 1 / (a[i] + b[i])
   }
})
# list of fixed constants
constants \leftarrow list(G = 2,
                 n = matrix(c(13, 12, 12, 11, 9, 10, 9, 9, 8, 11, 8, 10, 13,
                     10, 12, 9, 10, 9, 10, 5, 9, 9, 13, 7, 5, 10, 7, 6,
                     10, 10, 10, 7), nrow = 2)
# list specifying model data
data \leftarrow list(r = matrix(c(13, 12, 12, 11, 9, 10, 9, 9, 8, 10, 8, 9, 12, 9,
                11, 8, 9, 8, 9, 4, 8, 7, 11, 4, 4, 5, 5, 3, 7, 3,
                7, 0), nrow = 2))
# list specifying initial values
inits \leftarrow list(a = c(1, 1),
             b = c(1, 1),
             p = matrix(0.5, nrow = 2, ncol = 16),
             mu = c(.5, .5),
             theta = c(.5, .5))
# build the R model object
Rmodel <- nimbleModel(litters_code,</pre>
                     constants = constants,
                     data = data,
                             = inits)
                     inits
##### MCMC configuration and building #####
```

```
# generate the default MCMC configuration;
# only wish to monitor the derived quantity "mu"
mcmcConf <- configureMCMC(Rmodel, monitors = "mu")</pre>
# check the samplers assigned by default MCMC configuration
mcmcConf$printSamplers()
# double-check our monitors, and thinning interval
mcmcConf$printMonitors()
# build the executable R MCMC function
mcmc <- buildMCMC(mcmcConf)</pre>
# let's try another MCMC, as well,
# this time using the crossLevel sampler for top-level nodes
# generate an empty MCMC configuration
# we need a new copy of the model to avoid compilation errors
Rmodel2 <- Rmodel$newModel()</pre>
mcmcConf CL <- configureMCMC(Rmodel2, nodes = NULL, monitors = "mu")</pre>
# add two crossLevel samplers
mcmcConf_CL$addSampler(target = c("a[1]", "b[1]"), type = "crossLevel")
mcmcConf_CL$addSampler(target = c("a[2]", "b[2]"), type = "crossLevel")
# let's check the samplers
mcmcConf_CL$printSamplers()
# build this second executable R MCMC function
mcmc_CL <- buildMCMC(mcmcConf_CL)</pre>
######################################
##### compile to C++, and run #####
######################################
# compile the two copies of the model
Cmodel <- compileNimble(Rmodel)</pre>
Cmodel2 <- compileNimble(Rmodel2)</pre>
# compile both MCMC algorithms, in the same
# project as the R model object
# NOTE: at this time, we recommend compiling ALL
# executable MCMC functions together
Cmcmc <- compileNimble(mcmc, project = Rmodel)</pre>
Cmcmc_CL <- compileNimble(mcmc_CL, project = Rmodel2)</pre>
```

```
# run the default MCMC function,
# and example the mean of mu[1]
Cmcmc$run(1000)
cSamplesMatrix <- as.matrix(Cmcmc$mvSamples) # alternative: as.list
mean(cSamplesMatrix[, "mu[1]"])
# run the crossLevel MCMC function,
# and examine the mean of mu[1]
Cmcmc_CL$run(1000)
cSamplesMatrix_CL <- as.matrix(Cmcmc_CL$mvSamples)
mean(cSamplesMatrix_CL[, "mu[1]"])
######################################
#### run multiple MCMC chains #####
#####################################
# run 3 chains of the crossLevel MCMC
samplesList <- runMCMC(Cmcmc CL, niter=1000, nchains=3)</pre>
lapply(samplesList, dim)
```

7.12 Comparing different MCMCs with MCMCsuite and compareMCMCs

Please see the compareMCMCs package for the features previously provided by MCMCsuite and compareMCMCs in NIMBLE (until version 0.8.0). The compareMCMCs package provides tools to automatically run MCMC in nimble (including multiple sampler configurations), WinBUGS, Open-BUGS, JAGS, Stan, or any other engine for which you provide a simple common interface. The package makes it easy to manage comparison metrics and generate html pages with comparison figures.

7.13 Running MCMC chains in parallel

It is possible to run multiple chains in parallel using standard R parallelization packages such as parallel, foreach, and future. However, you must create separate copies of all model and MCMC objects using nimbleModel, buildMCMC, compileNimble, etc. This is because NIMBLE uses Reference Classes and R6 classes, so copying such objects simply creates a new variable name that refers to the original object.

Thus, in your parallel loop or lapply-style statement, you should run nimbleModel and all subsequent calls to create and compile the model and MCMC algorithm within the parallelized block of code, once for each MCMC chain being run in parallel.

For a worked example, please see the parallelization example on our webpage.

Chapter 8

Sequential Monte Carlo, Particle MCMC, Iterated Filtering, and MCEM

The NIMBLE algorithm library is growing and currently includes a suite of sequential Monte Carlo (particle filtering) algorithms, particle MCMC for combining particle filters with MCMC, iterated filtering version 2 and Monte Carlo expectation maximization (MCEM) for maximum likelihood estimation, and k-fold cross-validation.

8.1 Particle filters / sequential Monte Carlo and iterated filtering

As of Version 0.10.0 of NIMBLE, all of NIMBLE's sequential Monte Carlo/particle filtering functionality lives in the nimbleSMC package. Please load this package before trying to use these algorithms.

8.1.1 Filtering algorithms

NIMBLE includes algorithms for four different types of sequential Monte Carlo (also known as particle filters), which can be used to sample from the latent states and approximate the log likelihood of a state-space model. These include the bootstrap filter, the auxiliary particle filter, the Liu-West filter, and the ensemble Kalman filter. The iterated filtering version 2 (IF2) is a related method for maximum-likelihood estimation. Each of these is built with the eponymous functions buildBootstrapFilter, buildAuxiliaryFilter, buildLiuWestFilter, buildEnsembleKF, and buildIteratedFilter2. Each method requires setup arguments model and nodes; the latter should be a character vector specifying latent model nodes. In addition, each method can be customized using a control list argument. Details on the control options and specifics of the algorithms can be found in the help pages for the functions.

Once built, each filter can be run by specifying the number of particles. Each filter has a modelValues object named mvEWSamples that is populated with equally-weighted samples from the posterior distribution of the latent states (and in the case of the Liu-West filter, the posterior distribution of the top level parameters as well) as the filter is run. The bootstrap, auxiliary, and Liu-West filters, as well as the IF2 method, also have another modelValues object, mvWSamples. This has

unequally-weighted samples from the posterior distribution of the latent states, along with weights for each particle. In addition, the bootstrap and auxiliary particle filters return estimates of the log-likelihood of the given state-space model.

We first create a linear state-space model to use as an example for our particle filter algorithms.

```
# Building a simple linear state-space model.
# x is latent space, y is observed data
timeModelCode <- nimbleCode({</pre>
  x[1] ~ dnorm(mu_0, 1)
  y[1] \sim dnorm(x[1], 1)
  for(i in 2:t){
    x[i] \sim dnorm(x[i-1] * a + b, 1)
    y[i] \sim dnorm(x[i] * c, 1)
  }
  a \sim dunif(0, 1)
  b \sim dnorm(0, 1)
  c \sim dnorm(1,1)
  mu_0 ~ dnorm(0, 1)
})
# simulate some data
t <- 25; mu_0 <- 1
x <- rnorm(1 ,mu_0, 1)
y \leftarrow rnorm(1, x, 1)
a <- 0.5; b <- 1; c <- 1
for(i in 2:t){
 x[i] \leftarrow rnorm(1, x[i-1] * a + b, 1)
  y[i] \leftarrow rnorm(1, x[i] * c, 1)
}
# build the model
rTimeModel <- nimbleModel(timeModelCode, constants = list(t = t),
                            data <- list(y = y), check = FALSE )</pre>
# Set parameter values and compile the model
rTimeModel$a <- 0.5
rTimeModel$b <- 1
rTimeModel$c <- 1
rTimeModel$mu_0 <- 1
cTimeModel <- compileNimble(rTimeModel)</pre>
```

8.1.1.1 Bootstrap filter

Here is an example of building and running the bootstrap filter.

8.1.1.2 Auxiliary particle filter

Next, we provide an example of building and running the auxiliary particle filter. Note that a filter cannot be built on a model that already has a filter specialized to it, so we create a new copy of our state space model first.

8.1.1.3 Liu and West filter

Now we give an example of building and running the Liu and West filter, which can sample from the posterior distribution of top-level parameters as well as latent states. Note that the Liu-West filter ofen performs poorly and is provided primarily for didactic purposes. The Liu and West filter accepts an additional params argument, specifying the top-level parameters to be sampled.

```
## Warning in buildLiuWestFilter(LWTimeModel, "x", params = c("a", "b", "c"), :
## The Liu-West filter ofen performs poorly and is provided primarily for didactic
## purposes.

# Compile filter
cLWF <- compileNimble(rLWF,project = LWTimeModel)
# Run Liu-West filter
cLWF$run(parNum)</pre>
```

8.1.1.4 Ensemble Kalman filter

Next we give an example of building and running the ensemble Kalman filter, which can sample from the posterior distribution of latent states.

Once each filter has been run, we can extract samples from the posterior distribution of our latent states as follows:

```
# Equally-weighted samples (available from all filters)
bootEWSamp <- as.matrix(cBootF$mvEWSamples) # alternative: as.list
auxEWSamp <- as.matrix(cAuxF$mvEWSamples)
LWFEWSamp <- as.matrix(cLWF$mvEWSamples)
ENKFEWSamp <- as.matrix(cENKF$mvEWSamples)

# Unequally-weighted samples, along with weights (available
# from bootstrap, auxiliary, and Liu and West filters)
bootWSamp <- as.matrix(cBootF$mvWSamples, "x")
bootWts <- as.matrix(cBootF$mvWSamples, "wts")
auxWSamp <- as.matrix(xAuxF$mvWSamples, "x")
auxWts <- as.matrix(cAuxF$mvWSamples, "wts")

# Liu and West filter also returns samples
# from posterior distribution of top-level parameters:
aEWSamp <- as.matrix(cLWF$mvEWSamples, "a")</pre>
```

8.1.1.5 Iterated filtering 2 (IF2)

The IF2 method accomplishes maximum likelihood estimation using a scheme wherein both latent states and parameters are represented by particles that are weighted and resampled during the iterations. Iterations include perturbations to the parameter particles following a schedule of decreasing magnitude to yield convergence to the MLE.

Here we apply IF2 to Nile River flow data, specifying a changepoint in the year the Aswan Dam was constructed, as the dam altered river flows.

```
library(FKF)
flowCode <- nimbleCode({</pre>
    for(t in 1:n)
        y[t] ~ dnorm(x[t], sd = sigmaMeasurements)
    x[1] \sim dnorm(x0, sd = sigmaInnovations)
    for(t in 2:n)
        x[t] \sim dnorm((t-1==28)*meanShift1899 + x[t-1], sd = sigmaInnovations)
    logSigmaInnovations ~ dnorm(0, sd = 100)
    logSigmaMeasurements ~ dnorm(0, sd = 100)
    sigmaInnovations <- exp(logSigmaInnovations)</pre>
    sigmaMeasurements <- exp(logSigmaMeasurements)</pre>
    x0 \sim dnorm(1120, var = 100)
    meanShift1899 ~ dnorm(0, sd = 100)
})
flowModel <- nimbleModel(flowCode, data = list(y = Nile),</pre>
                  constants = list(n = length(Nile)),
                  inits = list(logSigmaInnovations = log(sd(Nile)),
                                logSigmaMeasurements = log(sd(Nile)),
                                meanShift1899 = -100)
```

Note that the prior distributions for the parameters are not used by IF2, except possibly to obtain boundaries of valid parameter values (not the case here).

Now we build the filter, specifying user-controlled standard deviations (in this case the same as the perturbation sigma values) for use in generating the initial particles for the parameters via the control list.

We now run the algorithm with 1000 particles for 100 iterations with the schedule parameter equal to 0.2.

In addition to the estimates, we can extract the values of the log-likelihood, the estimates and the standard deviation of the parameter particles as they evolve over the iterations, in order to assess convergence.

```
set.seed(1)
est \leftarrow cFilter$run(m = 1000, niter = 100, alpha = 0.2)
cFilter$estimates[95:100,] ## Last 5 iterations of parameter values
##
                [,1]
                         [,2]
                                    [,3]
## [1,] 0.013306153 4.822613 -269.7837
## [2,] -0.013961938 4.857470 -271.4965
## [3,] 0.002183997 4.850475 -271.1919
## [4,] 0.015598044 4.851961 -272.3889
## [5,] -0.005571944 4.842719 -272.6390
## [6,] 0.042982317 4.837347 -271.1800
cFilter$logLik[90:100] ## Last 5 iterations of log likelihood values
##
    [1] -627.0497 -626.9570 -626.9856 -626.9954 -626.7448 -626.7521 -626.8472
    [8] -626.7398 -626.9869 -627.1354 -626.6648
```

Comparing to use of the Kalman Filter from the FKF package, we see the log-likelihood is fairly similar:

[1] -626.5521

8.1.2 Particle MCMC (PMCMC)

Particle MCMC (PMCMC) is a method that uses MCMC for top-level model parameters and uses a particle filter to approximate the time-series likelihood for use in determining MCMC acceptance probabilities (Andrieu et al., 2010). NIMBLE implements PMCMC by providing random-walk Metropolis-Hastings samplers for model parameters that make use of particle filters in this way. These samplers can use NIMBLE's bootstrap filter or auxiliary particle filter, or they can use a user-defined filter. Whichever filter is specified will be used to obtain estimates of the likelihood of the state-space model (marginalizing over the latent states), which is used for calculation of the Metropolis-Hastings acceptance probability. The RW_PF sampler uses a univariate normal proposal distribution, and can be used to sample scalar top-level parameters. The RW_PF_block sampler uses a multivariate normal proposal distribution and can be used to jointly sample vectors of top-level parameters. The PMCMC samplers can be specified with a call to addSampler with type

= "RW_PF" or type = "RW_PF_block", a syntax similar to the other MCMC samplers listed in Section 7.10.

The RW_PF sampler and RW_PF_block sampler can be customized using the control list argument to set the adaptive properties of the sampler and options for the particle filter algorithm to be used. In addition, providing pfOptimizeNparticles=TRUE in the control list will use an experimental algorithm to estimate the optimal number of particles to use in the particle filter. See help(samplers) for details. The MCMC configuration for the timeModel in the previous section will serve as an example for the use of our PMCMC sampler. Here we use the identity matrix as our proposal covariance matrix.

The type = "RW_PF" and type = "RW_PF*block" samplers default to using a bootstrap filter. The adaptation control parameters adaptive, adaptInterval, and adaptScaleOnly work in the same way as for an RW and RW*block samplers. However, it is not clear if the same approach to adaptation works well for PMCMC, so one should consider turning off adaptation and using a well-chosen proposal covariance.

It is also possible that more efficient results can be obtained by using a custom filtering algorithm. Choice of filtering algorithm can be controlled by the pfType control list entry. The pfType entry can be set either to 'bootstrap' (the default), 'auxiliary', or the name of a user-defined nimbleFunction that returns a likelihood approximation.

Any user-defined filtering nimbleFunction named in the pfType control list entry must satisfy the following:

- 1. The nimbleFunction must be the result of a call to nimbleFunction().
- 2. The nimbleFunction must have setup code that accepts the following (and only the following) arguments:
 - model, the NIMBLE model object that the MCMC algorithm is defined on.
 - latents, a character vector specifying the latent model nodes over which the particle filter will stochastically integrate over to estimate the log-likelihood function.
 - control, an R list object. Note that the control list can be used to pass in any additional information or arguments that the custom filter may require.
- 3. The nimbleFunction must have a run function that accepts a single integer arugment (the number of particles to use), and returns a scalar double (the log-likelihood estimate).
- 4. The nimbleFunction must define, in setup code, a modelValues object named mvEWSamples that is used to contain equally weighted samples of the latent states (that is, the latents argument to the setup function). Each time the run() method of the nimbleFunction is called with number of particles m, the mvEWSamples modelValues object should be resized to be of size m via a call to resize(mvEWSamples, m).

8.2 Monte Carlo Expectation Maximization (MCEM)

Suppose we have a model with missing data (or a layer of latent variables that can be treated as missing data), and we would like to maximize the marginal likelihood of the model, integrating over the missing data. A brute-force method for doing this is MCEM. This is an EM algorithm in which the missing data are simulated via Monte Carlo (often MCMC, when the full conditional distributions cannot be directly sampled from) at each iteration. MCEM can be slow, and there are other methods for maximizing marginal likelihoods that can be implemented in NIMBLE. The reason we started with MCEM is to explore the flexibility of NIMBLE and illustrate the ability to combine R and NIMBLE to run an algorithm, with R managing the highest-level processing of the algorithm and calling nimbleFunctions for computations.

NIMBLE provides an ascent-based MCEM algorithm, created using buildMCEM, that automatically determines when the algorithm has converged by examining the size of the changes in the likelihood between each iteration. Additionally, the MCEM algorithm can provide an estimate of the asymptotic covariance matrix of the parameters. An example of calculating the asymptotic covariance can be found in Section 8.2.1.

We will revisit the pump example to illustrate the use of NIMBLE's MCEM algorithm.

The first argument buildMCEM, model, is a NIMBLE model, which can be either the uncompiled or compiled version. At the moment, the model provided cannot be part of another MCMC sampler.

Initial values for the parameters are taken to be the values in the model at the time buildMCEM is called, unless the values in the compiled model are changed before running the MCEM.

The ascent-based MCEM algorithm has a number of control options:

The latentNodes argument should indicate the nodes that will be integrated over (sampled via MCMC), rather than maximized. These nodes must be stochastic, not deterministic! latentNodes will be expanded as described in Section 13.3.1.1. I.e., either latentNodes = "x" or latentNodes = c("x[1]", "x[2]") will treat x[1] and x[2] as latent nodes if x is a vector of two values. All other non-data nodes will be maximized over. Note that latentNodes can include discrete nodes, but the nodes to be maximized cannot.

The burnIn argument indicates the number of samples from the MCMC for the E-step that should be discarded when computing the expected likelihood in the M-step. Note that burnIn can be set to values lower than in standard MCMC computations, as each iteration will start where the last left off.

The mcmcControl argument will be passed to configureMCMC to define the MCMC to be used.

The MCEM algorithm automatically detects box constraints for the nodes that will be optimized, using NIMBLE's getBounds function. It is also possible for a user to manually specify constraints via the boxConstraints argument. Each constraint given should be a list in which the first element is the names of the nodes or variables that the constraint will be applied to and the second element is a vector of length two, in which the first value is the lower limit and the second is the upper limit. Values of Inf and -Inf are allowed. If a node is not listed, its constraints will be automatically determined by NIMBLE. These constraint arguments are passed as the lower and upper arguments to R's optim function, using method = "L-BFGS-B". Note that NIMBLE will give a warning if a user-provided constraint is more extreme than the constraint determined by NIMBLE.

The value of the buffer argument shrinks the boxConstraints by this amount. This can help protect against non-finite values occurring when a parameter is on the boundary.

In addition, the MCEM has some extra control options that can be used to further tune the convergence criterion. See help(buildMCEM) for more information.

The buildMCEM function returns a list with two elements. The first element is a function called run, which will use the MCEM algorithm to estimate the MLEs. The second function is called estimateCov, and is described in Section 8.2.1. The run function can be run as follows. There is only one run-time argument, initM, which is the number of MCMC iterations to use when the algorithm is initialized.

```
pumpMLE <- pumpMCEM$run(initM = 1000)</pre>
```

```
## Iteration Number: 1.
## Current number of MCMC iterations: 1000.
## Parameter Estimates:
##
       alpha
                  beta
## 0.8219803 1.1492317
## Convergence Criterion: 1.001.
## Iteration Number: 2.
## Current number of MCMC iterations: 1000.
## Parameter Estimates:
##
      alpha
                beta
## 0.818586 1.237039
## Convergence Criterion: 0.02393763.
## Iteration Number: 3.
## Current number of MCMC iterations: 1000.
## Parameter Estimates:
##
       alpha
                  beta
## 0.8274042 1.2673636
## Convergence Criterion: 0.002330748.
## Iteration Number: 4.
## Current number of MCMC iterations: 3845.
## Parameter Estimates:
##
       alpha
                  beta
## 0.8270797 1.2730042
## Convergence Criterion: 0.0002820885.
```

```
pumpMLE
```

```
## alpha beta
## 0.8270797 1.2730042
```

Direct maximization after analytically integrating over the latent nodes (possible for this model but often not feasible) gives estimates of $\hat{\alpha} = 0.823$ and $\hat{\beta} = 1.261$, so the MCEM seems to do pretty well, though tightening the convergence criteria may be warranted in actual usage.

8.2.1 Estimating the asymptotic covariance From MCEM

The second element of the list returned by a call to buildMCEM is a function called estimateCov, which estimates the asymptotic covariance of the parameters at their MLE values. If the run function has been called previously, the estimateCov function will automatically use the MLE values produced by the run function to estimate the covariance. Alternatively, a user can supply their own MLE values using the MLEs argument, which allows the covariance to be estimated without having called the run function. More details about the estimateCov function can be found by calling help(buildMCEM). Below is an example of using the estimateCov function.

```
pumpCov <- pumpMCEM$estimateCov()
pumpCov

## alpha beta
## alpha 0.1283888 0.2180972
## beta 0.2180972 0.6377524

# Alternatively, you can manually specify the MLE values as a named vector.
pumpCov <- pumpMCEM$estimateCov(MLEs = c(alpha = 0.823, beta = 1.261))</pre>
```

Chapter 9

Spatial models

NIMBLE supports two variations of conditional autoregressive (CAR) model structures: the improper intrinsic Gaussian CAR (ICAR) model, and a proper Gaussian CAR model. This includes distributions to represent these spatially-dependent model structures in a BUGS model, as well as specialized MCMC samplers for these distributions.

9.1 Intrinsic Gaussian CAR model: dcar normal

The intrinsic Gaussian conditional autoregressive (ICAR) model used to model dependence of block-level values (e.g., spatial areas or temporal blocks) is implemented in NIMBLE as the dcar_normal distribution. Additional details for using this distribution are available using help('CAR-Normal').

ICAR models are improper priors for random fields (e.g., temporal or spatial processes). The prior is a joint prior across a collection of latent process values. For more technical details on CAR models, including higher-order CAR models, please see Rue and Held (2005), Banerjee et al. (2015), and Paciorek (2009). Since the distribution is improper it should not be used as the distribution for data values, but rather to specify a prior for an unknown process. As discussed in the references above, the distribution can be seen to be a proper density in a reduced dimension subspace; thus the impropriety only holds on one or more linear combinations of the latent process values.

In addition to our focus here on CAR modeling for spatial data, the ICAR model can also be used in other contexts, such as for temporal data in a discrete time context.

9.1.1 Specification and density

NIMBLE uses the same parameterization as WinBUGS / GeoBUGS for the dcar_normal distribution, providing compatibility with existing WinBUGS code. NIMBLE also provides the WinBUGS name car.normal as an alias.

9.1.1.1 Specification

The dcar normal distribution is specified for a set of N spatially dependent regions as:

x[1:N] ~ dcar_normal(adj, weights, num, tau, c, zero_mean)

The adj, weights and num parameters define the adjacency structure and associated weights of the spatially-dependent field. See help('CAR-Normal') for details of these parameters. When specifying a CAR distribution, these parameters must have constant values. They do not necessarily have to be specified as constants when creating a model object using nimbleModel, but they should be defined in a static way: as right-hand-side only variables with initial values provided as constants, data or inits, or using fixed numerical deterministic declarations. Each of these two approaches for specifying values are shown in the example.

The adjacency structure defined by adj and the associated weights must be symmetric. That is, if region i is neighbor of region j, then region j must also be a neighbor of region i. Further, the weights associated with these reciprocating relationships must be equal. NIMBLE performs a check of these symmetries and will issue an error message if asymmetry is detected.

The scalar precision tau may be treated as an unknown model parameter and itself assigned a prior distribution. Care should be taken in selecting a prior distribution for tau, and WinBUGS suggests that users be prepared to carry out a sensitivity analysis for this choice.

When specifying a higher-order CAR process, the number of constraints c can be explicitly provided in the model specification. This would be the case, for example, when specifying a thin-plate spline (second-order) CAR model, for which c should be 2 for a one-dimensional process and 3 for a two-dimensional (e.g., spatial) process, as discussed in Rue and Held (2005) and Paciorek (2009). If c is omitted, NIMBLE will calculate c as the number of disjoint groups of regions in the adjacency structure, which implicitly assumes a first-order CAR process for each group.

By default there is no zero-mean constraint imposed on the CAR process, and thus the mean is implicit within the CAR process values, with an implicit improper flat prior on the mean. To avoid non-identifiability, one should not include an additional parameter for the mean (e.g., do not include an intercept term in a simple CAR model with first-order neighborhood structure). When there are disjoint groups of regions and the constraint is not imposed, there is an implicit distinct improper flat prior on the mean for each group, and it would not make sense to impose the constraint since the constraint holds across all regions. Similarly, if one sets up a neighborhood structure for higher-order CAR models, it would not make sense to impose the zero-mean constraint as that would account for only one of the eigenvalues that are zero. Imposing this constraint (by specifying the parameter zero_mean = 1) allows users to model the process mean separately, and hence a separate intercept term should be included in the model.

NIMBLE provides a convenience function as.carAdjacency for converting other representations of the adjacency information into the required adj, weights, num format. This function can convert:

- A symmetric adjacency matrix of weights (with diagonal elements equal to zero), using as.carAdjacency(weightMatrix)
- Two length-N lists with numeric vector elements giving the neighboring indices and associated weights for each region, using as.carAdjacency(neighborList, weightList)

These conversions should be done in R, and the resulting adj, weights, num vectors can be passed as constants into nimbleModel.

9.1.1.2 Density

For process values $x = (x_1, \dots, x_N)$ and precision τ , the improper CAR density is given as:

$$p(x| au) \propto au^{(N-c)/2} \; e^{-rac{ au}{2} \sum_{i
eq j} w_{ij} \, (x_i - x_j)^2}$$

where the summation over all (i, j) pairs, with the weight between regions i and j given by w_{ij} , is equivalent to summing over all pairs for which region i is a neighbor of region j. Note that the value of c modifies the power to which the precision is raised, accounting for the impropriety of the density based on the number of zero eigenvalues in the implicit precision matrix for x.

For the purposes of MCMC sampling the individual CAR process values, the resulting conditional prior of region i is:

$$p(x_i|x_{-i},\tau) \sim \mathcal{N}\left(\frac{1}{w_{i+}} \sum_{j \in \mathcal{N}_i} w_{ij} \, x_j, \ w_{i+}\tau\right)$$

where x_{-i} represents all elements of x except x_i , the neighborhood \mathcal{N}_i of region i is the set of all j for which region j is a neighbor of region i, $w_{i+} = \sum_{j \in \mathcal{N}_i} w_{ij}$, and the Normal distribution is parameterized in terms of precision.

9.1.2 Example

Here we provide an example model using the intrinsic Gaussian dcar_normal distribution. The CAR process values are used in a spatially-dependent Poisson regression.

To mimic the behavior of WinBUGS, we specify zero_mean = 1 to enforce a zero-mean constraint on the CAR process, and therefore include a separate intercept term alpha in the model. Note that we do not necessarily recommend imposing this constraint, per the discussion earlier in this chapter.

```
code <- nimbleCode({</pre>
    alpha ~ dflat()
    beta ~ dnorm(0, 0.0001)
    tau \sim dgamma(0.001, 0.001)
    s[1:N] ~ dcar_normal(adj[1:L], weights[1:L], num[1:N], tau, zero_mean = 1)
    for(i in 1:N) {
        log(lambda[i]) <- alpha + beta*x[i] + s[i]</pre>
        y[i] ~ dpois(lambda[i])
    }
})
L <- 8
constants \leftarrow list(N = 4, L = L, num = c(3, 2, 2, 1), weights = rep(1, L),
                   adj = c(2,3,4,1,3,1,2,1), x = c(0, 2, 2, 8))
data \leftarrow list(y = c(6, 9, 7, 12))
inits \leftarrow list(alpha = 0, beta = 0, tau = 1, s = c(0, 0, 0, 0))
Rmodel <- nimbleModel(code, constants, data, inits)</pre>
```

The resulting model may be carried through to MCMC sampling. NIMBLE will assign a specialized sampler to the update the elements of the CAR process. See Chapter 7 for information about NIMBLE's MCMC engine, and Section 9.3 for details on MCMC sampling of the CAR processes.

9.2 Proper Gaussian CAR model: dcar_proper

The proper Gaussian conditional autoregressive model used to model dependence of block-level values (e.g., spatial areas or temporal blocks) is implemented in NIMBLE as the dcar_proper distribution. Additional details of using this distribution are available using help('CAR-Proper').

Proper CAR models are proper priors for random fields (e.g., temporal or spatial processes). The prior is a joint prior across a collection of latent process values. For more technical details on proper CAR models please see Banerjee et al. (2015), including considerations of why the improper CAR model may be preferred.

In addition to our focus here on CAR modeling for spatial data, the proper CAR model can also be used in other contexts, such as for temporal data in a discrete time context.

9.2.1 Specification and density

NIMBLE uses the same parameterization as WinBUGS / GeoBUGS for the dcar_proper distribution, providing compatibility with existing WinBUGS code. NIMBLE also provides the WinBUGS name car.proper as an alias.

9.2.1.1 Specification

The dcar proper distribution is specified for a set of N spatially dependent regions as:

x[1:N] ~ dcar_proper(mu, C, adj, num, M, tau, gamma)

There is no option of a zero-mean constraint for proper CAR process, and instead the mean for each region is specified by the mu parameter. The elements of mu can be assigned fixed values or may be specified using one common, or multiple, prior distributions.

The C, adj, num and M parameters define the adjacency structure, normalized weights, and conditional variances of the spatially-dependent field. See help('CAR-Proper') for details of these parameters. When specifying a CAR distribution, these parameters must have constant values. They do not necessarily have to be specified as constants when creating a model object using nimbleModel, but they should be defined in a static way: as right-hand-side only variables with initial values provided as constants, data or inits, or using fixed numerical deterministic declarations.

The adjacency structure defined by adj must be symmetric. That is, if region i is neighbor of region j, then region j must also be a neighbor of region i. In addition, the normalized weights specified in C must satisfy a symmetry constraint jointly with the conditional variances given in M. This constraint requires that $M^{-1}C$ is symmetric, where M is a diagonal matrix of conditional variances and C is the normalized (each row sums to one) weight matrix. Equivalently, this implies that $C_{ij}M_{jj}=C_{ji}M_{ii}$ for all pairs of neighboring regions i and j. NIMBLE performs a check of these symmetries and will issue an error message if asymmetry is detected.

Two options are available to simplify the process of constructing the C and M arguments; both options are demonstrated in the example. First, these arguments may be omitted from the dcar_proper specification. In this case, values of C and M will be generated that correspond to all weights being equal to one, or equivalently, a symmetric weight matrix containing only zeros and ones. Note that C and M should either both be provided, or both be omitted from the specification.

Second, a convenience function as.carCM is provided to generate the C and M arguments corresponding to a specified set of symmetric unnormalized weights. If weights contains the non-zero weights corresponding to an unnormalized weight matrix (weights is precisely the argument that can be used in the dcar_normal specification), then a list containing C and M can be generated using as.carCM(adj, weights, num). In this case, the resulting C contains the row-normalized weights, and the resulting M is a vector of the inverse row-sums of the unnormalized weight matrix.

The scalar precision tau may be treated as an unknown model parameter and itself assigned a prior distribution. Care should be taken in selecting a prior distribution for tau, and WinBUGS suggests that users be prepared to carry out a sensitivity analysis for this choice.

An appropriate value of the gamma parameter ensures the propriety of the dcar_proper distribution. The value of gamma must lie between fixed bounds, which are given by the reciprocals of the largest and smallest eigenvalues of $M^{-1/2}CM^{1/2}$. These bounds can be calculated using the function carBounds or separately using the functions carMinBound and carMaxBound. For compatibility with WinBUGS, NIMBLE provides min.bound and max.bound as aliases for carMinBound and carMaxBound. Rather than selecting a fixed value of gamma within these bounds, it is recommended that gamma be assigned a uniform prior distribution over the region of permissible values.

Note that when C and M are omitted from the dcar_proper specification (and hence all weights are taken as one), or C and M are calculated from a symmetric weight matrix using the utility function as.carCM, then the bounds on gamma are necessarily (-1,1). In this case, gamma can simply be assigned a prior over that region. This approach is shown in both examples.

9.2.1.2 Density

The proper CAR density is given as:

$$p(x|\mu, C, M, \tau, \gamma) \sim \text{MVN}\left(\mu, \frac{1}{\tau}(I - \gamma C)^{-1}M\right)$$

where the multivariate normal distribution is parameterized in terms of covariance.

For the purposes of MCMC sampling the individual CAR process values, the resulting conditional prior of region i is:

$$p(x_i|x_{-i},\mu,C,M,\tau,\gamma) \sim \mathrm{N}\left(\mu_i + \textstyle\sum_{j \in \mathcal{N}_i} \gamma \, C_{ij} \, (x_j - \mu_j), \,\, \frac{M_{ii}}{\tau}\right)$$

where x_{-i} represents all elements of x except x_i , the neighborhood \mathcal{N}_i of region i is the set of all j for which region j is a neighbor of region i, and the Normal distribution is parameterized in terms of variance.

9.2.2 Example

We provide two example models using the proper Gaussian dcar_proper distribution. In both, the CAR process values are used in a spatially-dependent logistic regression to model binary presence/absence data. In the first example, the C and M parameters are omitted, which uses weights equal to one for all neighbor relationships. In the second example, symmetric unnormalized weights are specified, and as.carCM is used to construct the C and M parameters to the dcar_proper distribution.

```
# omitting C and M sets all non-zero weights to one
code <- nimbleCode({</pre>
    mu0 ~ dnorm(0, 0.0001)
    tau \sim dgamma(0.001, 0.001)
    gamma \sim dunif(-1, 1)
    s[1:N] ~ dcar_proper(mu[1:N], adj=adj[1:L], num=num[1:N], tau=tau,
                           gamma=gamma)
    for(i in 1:N) {
        mu[i] <- muO
        logit(p[i]) <- s[i]</pre>
        y[i] ~ dbern(p[i])
    }
})
adj \leftarrow c(2, 1, 3, 2, 4, 3)
num \leftarrow c(1, 2, 2, 1)
constants \leftarrow list(adj = adj, num = num, N = 4, L = 6)
data \leftarrow list(y = c(1, 0, 1, 1))
inits \leftarrow list(mu0 = 0, tau = 1, gamma = 0, s = rep(0, 4))
Rmodel <- nimbleModel(code, constants, data, inits)</pre>
\# specify symmetric unnormalized weights, use as carCM to generate C and M
code <- nimbleCode({</pre>
    mu0 ~ dnorm(0, 0.0001)
    tau \sim dgamma(0.001, 0.001)
    gamma ~ dunif(-1, 1)
    s[1:N] ~ dcar_proper(mu[1:N], C[1:L], adj[1:L], num[1:N], M[1:N], tau,
                           gamma)
    for(i in 1:N) {
        mu[i] <- mu0
        logit(p[i]) <- s[i]</pre>
        y[i] ~ dbern(p[i])
    }
})
weights \leftarrow c(2, 2, 3, 3, 4, 4)
CM <- as.carCM(adj, weights, num)</pre>
constants <- list(C = CM$C, adj = adj, num = num, M = CM$M, N = 4, L = 6)
Rmodel <- nimbleModel(code, constants, data, inits)</pre>
```

Each of the resulting models may be carried through to MCMC sampling. NIMBLE will assign a specialized sampler to update the elements of the CAR process. See Chapter 7 for information about NIMBLE's MCMC engine, and Section 9.3 for details on MCMC sampling of the CAR processes.

9.3 MCMC Sampling of CAR models

NIMBLE's MCMC engine provides specialized samplers for the dcar_normal and dcar_proper distributions. These samplers perform sequential univariate updates on the components of the CAR process. Internally, each sampler assigns one of three specialized univariate samplers to each component, based on inspection of the model structure:

- 1. A conjugate sampler in the case of conjugate Normal dependencies.
- 2. A random walk Metropolis-Hastings sampler in the case of non-conjugate dependencies.
- 3. A posterior predictive sampler in the case of no dependencies.

Note that these univariate CAR samplers are not the same as NIMBLE's standard conjugate, RW, and posterior_predictive samplers, but rather specialized versions for operating on a CAR distribution. Details of these assignments are strictly internal to the CAR samplers.

In future versions of NIMBLE we expect to provide block samplers that update the entire CAR process as a single sample. This may provide improved MCMC performance by accounting for dependence between elements, particularly when conjugacy is available.

9.3.1 Initial values

Valid initial values should be provided for all elements of the process specified by a CAR structure before running an MCMC. This ensures that the conditional prior distribution is well-defined for each region. A simple and safe choice of initial values is setting all components of the process equal to zero, as is done in the preceding CAR examples.

For compatibility with WinBUGS, NIMBLE also allows an initial value of NA to be provided for regions with zero neighbors. This particular initialization is required in WinBUGS, so this allows users to make use of existing WinBUGS code.

9.3.2 Zero-neighbor regions

Regions with zero neighbors (defined by a 0 appearing in the num parameter) are a special case for both the dcar_normal and dcar_proper distribution. The corresponding neighborhood \mathcal{N} of such a region contains no elements, and hence the conditional prior is improper and uninformative, tantamount to a dflat prior distribution. Thus, the conditional posterior distribution of those regions is entirely determined by the dependent nodes, if any. Sampling of these zero-neighbor regions proceeds as:

- 1. In the conjugate case, sampling proceeds according to the conjugate posterior.
- 2. In the non-conjugate case, sampling proceeds using random walk Metropolis-Hastings, where the posterior is determined entirely by the dependencies.
- 3. In the case of no dependents, the posterior is entirely undefined. Here, no changes will be made to the process value, and it will remain equal to its initial value throughout. By virtue of having no neighbors, this region does not contribute to the density evaluation of the subsuming dcar_normal node nor to the conditional prior of any other regions, hence its value (even NA) is of no consequence.

This behavior is different from that of WinBUGS, where the value of zero-neighbor regions of car.normal nodes is set to and fixed at zero.

Also note that for the dcar_proper distribution if any regions have zero neighbors the joint density of the process cannot be calculated. As a result one cannot do MCMC sampling of unknown parameters affecting the mean (mu) of the process, though one can instead use an uncentered parameterization in which the mean is added to the process rather than the process being centered on the mean. Or one could remove such regions from the process and model them separately.

9.3.3 Zero-mean constraint

A zero-mean constraint is available for the intrinsic Gaussian dcar_normal distribution. This constraint on the ICAR process values is imposed during MCMC sampling, if the argument zero_mean = 1, mimicking the behavior of WinBUGS. Following the univariate updates on each component, the mean is subtracted away from all process values, resulting in a zero-mean process.

Note that this is *not* equivalent to sampling under the constraint that the mean is zero (see p. 36 of Rue and Held (2005)) so should be treated as an ad hoc approach and employed with caution.

Chapter 10

Bayesian nonparametric models

10.1 Bayesian nonparametric mixture models

NIMBLE provides support for Bayesian nonparametric (BNP) mixture modeling. The current implementation provides support for hierarchical specifications involving Dirichlet process (DP) mixtures (Ferguson, 1973, 1974; Lo, 1984; Escobar, 1994; Escobar and West, 1995). More specifically, a DP mixture model takes the form

$$\begin{aligned} y_i \mid G \overset{iid}{\sim} & \int h(y_i \mid \theta) G(d\theta), \\ & G \mid \alpha, G_0 \sim DP(\alpha, G_0), \end{aligned}$$

where $h(\cdot \mid \theta)$ is a suitable kernel with parameter θ , and α and G_0 are the concentration and baseline distribution parameters of the DP, respectively. DP mixture models can be written with different levels of hierarchy, all being equivalent to the model above.

When the random measure G is integrated out from the model, the DP mixture model can be written using latent or membership variables, z_i , following a Chinese Restaurant Process (CRP) distribution (Blackwell and MacQueen, 1973), discussed in Section 10.2. The model takes the form

$$\begin{split} y_i \mid \tilde{\theta}, z_i \overset{ind}{\sim} h(\cdot \mid \tilde{\theta}_{z_i}), \\ z \mid \alpha \sim \text{CRP}(\alpha), \quad \tilde{\theta}_i \overset{iid}{\sim} G_0, \end{split}$$

where $CRP(\alpha)$ denotes the CRP distribution with concentration parameter α .

If a stick-breaking representation (Sethuraman, 1994), discussed in section 10.3, is assumed for the random measure G, then the model takes the form

$$\begin{split} y_i \mid \boldsymbol{\theta}^{\star}, v \overset{ind}{\sim} \sum_{l=1}^{\infty} \left\{ v_l \prod_{m < l} (1 - v_m) \right\} h(\cdot \mid \boldsymbol{\theta}_l^{\star}), \\ v_l \mid \alpha \overset{iid}{\sim} Beta(1, \alpha), \quad \boldsymbol{\theta}_l^{\star} \overset{iid}{\sim} G_0. \end{split}$$

More general representations of the random measure can be specify by considering $v_l \mid \nu_l, \alpha_l \stackrel{ind}{\sim} Beta(\nu_l, \alpha_l)$. Finite dimensional approximations can be obtained by truncating the infinite sum to have L components.

Different representations of DP mixtures lead to different computational algorithms. NIMBLE supports sampling algorithms based on the CRP representation, as well as on the stick-breaking representation. NIMBLE includes definitions of structures required to implement the CRP and stick-breaking distributions, and the associated MCMC algorithms.

10.2 Chinese Restaurant Process model

The CRP is a distribution over the space of partitions of positive integers and is implemented in NIMBLE as the dCRP distribution. More details for using this distribution are available using help(CRP).

The CRP can be described as a stochastic process in which customers arrive at a restaurant, potentially with an infinite number of tables. Each customer sits at an empty or occupied table according to probabilities that depend on the number of customers in the occupied tables. Thus, the CRP partitions the set of customers, through their assignment to tables in the restaurant.

10.2.1 Specification and density

NIMBLE parametrizes the dCRP distribution by a concentration parameter and a size parameter.

10.2.1.1 Specification

The dCRP distribution is specified in NIMBLE for a membership vector z as

z[1:N] ~ dCRP(conc, size)

The conc parameter is the concentration parameter of the CRP, controlling the probability of a customer sitting on a new table, i.e., creating a new cluster. The size parameter defines the size of the set of integers to be partitioned.

The conc parameter is a positive real value that can be treated as known or unknown. When a gamma prior is assumed for the conc parameter, a specialized sampler is assigned. See more on this in section 10.4.1.

The size parameter is a positive integer that has to be fixed and equal to the length of vector z. It defines the set of consecutive integers from 1 to N to be partitioned. Each element in z can be an integer from 1 to N, and repetitions are allowed.

10.2.1.2 Density

The CRP distribution partitions the set of positive integers $1, \ldots, N$, into $N^* \leq N$ disjoint subsets, indicating to which subset each element belongs. For instance, if N=6, the set $\{1,2,3,4,5,6\}$ can be partitioned into the subsets $S_1=\{1,2,6\}$, $S_2=\{4,5\}$, and $S_3=\{3\}$. Note that $N^*=3$, and this is one partition from out of 203 possibilities. The CRP-distributed vector z encodes this partition and its observed values would be (1,1,3,2,2,1), for this example. In mixture modeling, this indicates that observations 1, 2, and 6 belong to cluster 1, observations 4 and 5 to cluster 2,

and observation 3 to cluster 3. Note that this representation is not unique, vector (2, 2, 1, 3, 3, 2) encodes the same partition.

The joint probability function of $z=(z_1,\ldots,z_N)$, with concentration parameter α , is given by

$$p(z\mid\alpha)\propto\frac{\Gamma(\alpha)}{\Gamma(\alpha+n)}\alpha^{N^{\star}(z)}\prod_{k=1}^{N^{\star}(z)}\Gamma(m_k(z)),$$

where $m_k(z)$ denotes the number of elements in z that are equal to k, $N^*(z)$ denotes the number of unique elements in z, and $\Gamma(\cdot)$ denotes the gamma function. The full conditional distribution for z_i given z_{-i} is

$$p(z_i = m \mid z_{-i}, \alpha) = \frac{1}{n-1+\alpha} \sum_{j \neq i} 1_{\{z_j\}}(m) + \frac{\alpha}{n-1+\alpha} 1_{\{z^{new}\}}(m),$$

where z_{-i} denotes vector z after removing its i—th component, z^{new} is a value not in z_{-i} , and 1_A denotes the indicator function at set A.

Note that the probability of creating a new cluster is proportional to α : the larger the concentration parameter, the more clusters are created.

10.2.2 Example

The following example illustrates how to use NIMBLE to perform single density estimation for real-valued data, under a BNP approach, using the dCRP distribution. (Note that the BNP approach is also often used to perform density estimation on random effects.) The model is given by

$$\begin{split} y_i \mid \tilde{\theta}, \tilde{\sigma}^2, z_i \overset{ind}{\sim} N(\tilde{\theta}_{z_i}, \tilde{\sigma}_{z_i}^2), & i = 1, \dots, N, \\ z \sim \text{CRP}(\alpha), & \alpha \sim \text{Gamma}(1, 1), \\ \tilde{\theta}_j \overset{iid}{\sim} N(0, 100), & \tilde{\sigma}_j^2 \overset{iid}{\sim} \text{InvGamma}(1, 1), \ j = 1, \dots, M. \end{split}$$

```
code <- nimbleCode({
    z[1:N] ~ dCRP(alpha, size = N)
    alpha ~ dgamma(1, 1)
    for(i in 1:M) {
        thetatilde[i] ~ dnorm(0, var = 100)
        s2tilde[i] ~ dinvgamma(1, 1)
    }
    for(i in 1:N)
        y[i] ~ dnorm(thetatilde[z[i]], var = s2tilde[z[i]])
})

set.seed(1)
constants <- list(N = 100, M = 50)
data <- list(y = c(rnorm(50, -5, sqrt(3)), rnorm(50, 5, sqrt(4))))
inits <- list(thetatilde = rnorm(constants$M, 0, 10),</pre>
```

```
s2tilde = rinvgamma(constants$M, 1, 1),
    z = sample(1:10, size = constants$N, replace = TRUE),
    alpha = 1)
model <- nimbleModel(code, constants, data, inits)</pre>
```

The model can be fitted through MCMC sampling. NIMBLE will assign a specialized sampler to update **z** and **alpha**. See Chapter 7 for information about NIMBLE's MCMC engine, and Section 10.4.1 for details on MCMC sampling of the CRP.

One of the advantages of BNP mixture models is that the number of clusters is treated as random. Therefore, in MCMC sampling, the number of cluster parameters varies with the iteration. Since NIMBLE does not currently allow dynamic length allocation, the number of unique cluster parameters, N^* , has to be fixed. One safe option is to set this number to N, but this is inefficient, both in terms of computation and in terms of storage, because in practice it is often that $N^* < N$. In addition, configuring and building the MCMC can be slow (and use a lot of memory) for large N. In an effort to mitigate these inefficiencies, we allow the user to set $N^* = M$, with M < N, as seen in the example above. However, if this number is too small and is exceeded in any iteration a warning is issued.

10.2.3 Extensions

The BNP functionality in NIMBLE was extended in version 0.10.0 to more general models. These extensions enable users, for instance, to use a DP or DPM prior for the distribution of the random effects in a generalized linear mixed effects model with mutiple measurements over time or multiple trials per participant.

The following example illustrates how to use NIMBLE in a random effects model with repeated measurements per subject using a DP prior for the distribution of the subject's random effects. The model is given by

$$\begin{split} y_{i,j} \mid \tilde{\theta}, z_i, \sigma^2 &\sim N(\tilde{\theta}_{z_i}, \sigma^2), \quad i = 1, \dots, N, \quad j = 1, \dots, J, \\ &z \sim \text{CRP}(\alpha), \quad \alpha \sim \text{Gamma}(1, 1), \\ &\tilde{\theta}_m \overset{iid}{\sim} N(0, 100), \ m = 1, \dots, M, \quad \sigma^2 \sim \text{InvGamma}(1, 1). \end{split}$$

The corresponding NIMBLE code is

```
code <- nimbleCode({
    z[1:N] ~ dCRP(alpha, size = N)
    alpha ~ dgamma(1, 1)
    sigma2 ~ dinvgamma(1, 1)
    for(i in 1:M) {
        thetatilde[i] ~ dnorm(0, var = 100)
    }
    for(i in 1:N) {
        for(j in 1:J) {</pre>
```

Alternatively, each subject could have a vector of parameters being clustered. For example in the model above one could instead specify a vector of means, such as thetaTilde[z[i], j], instead of a single mean. This allows group-specific parameters to also vary across the repeated mesasurements.

As before, the model can be fitted through MCMC sampling. NIMBLE will assign a specialized sampler to update **z** and **alpha**. See Chapter 7 for information about NIMBLE's MCMC engine, and Section 10.4.1 for details on MCMC sampling of the CRP.

10.3 Stick-breaking model

In NIMBLE, weights defined by sequentially breaking a stick, as in the stick-breaking process, are implemented as the stick_breaking link function. More details for using this function are available using help(stick_breaking).

10.3.1 Specification and function

NIMBLE parametrizes the stick_breaking function by vector of values in (0,1).

10.3.1.1 Function

The weights (w_1, \dots, w_L) follow a finite stick-breaking construction if

$$\begin{split} w_1 &= v_1,\\ w_l &= v_l \prod_{m < l} (1 - v_m), l = 2, \dots, L - 1\\ w_L &= \prod_{m < L} (1 - v_m). \end{split}$$

for
$$v_l \in [0, 1], l = 1, \dots, L - 1$$
.

10.3.1.2 Specification

The stick_breaking function is specified in NIMBLE for a vector w of probabilities as

```
w[1:L] <- stick_breaking(v[1:(L-1)])</pre>
```

The argument v is a vector of values between 0 and 1 defining the sequential breaking points of the stick after removing the previous portions already broken off. It is of length L-1, implicitly assuming that its last component is equal to 1.

In order to complete the definition of the weights in the stick-breaking representation of G, a prior distribution on (0,1) should to be assumed for v_l , $l=1,\ldots,L-1$, for instance a beta prior.

10.3.2 Example

Here we illustrate how to use NIMBLE for the example described in section 10.2.2, but considering a stick-breaking representation for G. The model is given by

```
\begin{split} y_i \mid \theta^{\star}, \sigma^{\star 2}, z_i \overset{ind}{\sim} N(\theta^{\star}_{z_i}, \sigma^{2^{\star}_{z_i}}), & i = 1, \dots, N, \\ z_i \sim \text{Categorical}(w), & i = 1, \dots, N, \\ v_l \overset{iid}{\sim} \text{Beta}(1, \alpha), & l = 1, \dots, L - 1, \\ & \alpha \sim \text{Gamma}(1, 1), \\ \theta^{\star}_l \overset{iid}{\sim} N(0, 100), & \sigma^{2^{\star}_l} \overset{iid}{\sim} \text{InvGamma}(1, 1), & l = 1, \dots, L. \end{split}
```

where $w_1=v_1, \ w_l=v_l\prod_{m< l}(1-v_m),$ for $l=1,\ldots,L-1,$ and $w_L=\prod_{m< L}(1-v_m).$

```
code <- nimbleCode({</pre>
  for(i in 1:(L-1)){
    v[i] ~ dbeta(1, alpha)
  }
  alpha ~ dgamma(1, 1)
  w[1:L] <- stick_breaking(v[1:(L-1)])</pre>
  for(i in 1:L) {
    thetastar[i] ~ dnorm(0, var = 100)
    s2star[i] ~ dinvgamma(1, 1)
  }
  for(i in 1:N) {
    z[i] \sim dcat(w[1:L])
    y[i] ~ dnorm(thetastar[z[i]], var = s2star[z[i]])
  }
})
set.seed(1)
constants \leftarrow list(N = 100, L=50)
data \leftarrow list(y = c(rnorm(50, -5, sqrt(3)), rnorm(50, 5, sqrt(4))))
inits <- list(thetastar = rnorm(constants$L, 0, 100),</pre>
               s2star = rinvgamma(constants$L, 1, 1),
               z = sample(1:10, size = constants$N, replace = TRUE),
```

```
v = rbeta(constants$L, 1, 1),
alpha = 1)
modelSB <- nimbleModel(code, constants, data, inits)</pre>
```

The resulting model may be carried through to MCMC sampling. NIMBLE will assign a specialized sampler to update v. See Chapter 7 for information about NIMBLE's MCMC engine, and Section 10.4.2 for details on MCMC sampling of the stick-breaking weights.

10.4 MCMC sampling of BNP models

BNP models can be specified in different, yet equivalent, manners. Examples 10.2.2 and 10.3.2 are examples of density estimation for real-valued data, and are specified through the CRP and the stick-breaking process, respectively. Different specifications lead NIMBLE to assign different sampling algorithms for the model. When the model is specified through a CRP, a collapsed sampler (Neal, 2000) is assigned. Under this specification, the random measure G is integrated out from the model. When a stick-breaking representation is used, a blocked Gibbs sampler is assigned, see Ishwaran and James (2001) and Ishwaran and James (2002).

10.4.1 Sampling CRP models

NIMBLE's MCMC engine provides specialized samplers for the dCRP distribution, updating each component of the membership vector sequentially. Internally, the sampler is assigned based on inspection of the model structure, evaluating conjugacy between the mixture kernel and the baseline distribution, as follows:

- 1. A conjugate sampler in the case of the baseline distribution being conjugate for the mixture kernel.
- 2. A non-conjugate sampler in the case of the baseline distribution not being conjugate for the mixture kernel.

Note that both samplers are specialized versions that operate on a vector having a CRP distribution. Details of these assignments are strictly internal to the CRP samplers. The current release of NIMBLE supports conjugate sampling for the dCRP distribution for the relationships listed in Table 10.1. Additional relationships are provided in Table 10.2 for normal mixture kernels when both mean and variance are unknown.

Table 10.1: Conjugate relationships for the dCRP distribution
supported by NIMBLE's MCMC engine.

Baseline Distribution	Mixture (Dependent Node) Distribution	Parameter
Beta	Bernoulli	prob
	Binomial	prob
	Negative Binomial	prob
Dirichlet	Multinomial	prob
Gamma	Poisson	lambda
	Normal	tau
	Gamma	rate
	Inverse Gamma	scale

Baseline Distribution	Mixture (Dependent Node) Distribution	Parameter
	Exponential	rate
	Weibull	lambda
Inverse Gamma	Normal	var
Normal	Normal	mean
Multivariate Normal	Multivariate Normal	mean
Wishart	Multivariate Normal	prec
Inverse Wishart	Multivariate Normal	cov

Table 10.2: Additional conjugate relationships for the dCRP distribution supported by NIMBLE's MCMC engine.

Baseline Distribution	Mixture (Dependent Node) Distribution	Parameter
Normal-Gamma	Normal	mean, tau
Normal-Inverse Gamma	Normal	mean, var
Multivariate Normal-Wishart	Multivariate Normal	mean, prec
Multivariate Normal-Inverse Wishart	Multivariate Normal	$\mathtt{mean},\mathtt{cov}$

To reduce computation and improve mixing, we only sample the parameters of the clusters (e.g., $\tilde{\theta}_j$ in 10.2.2 when the associated cluster is occupied, using the CRP_cluster_wrapper sampler, which wraps around an underlying actual sampler. In addition, this approach requires that any sampler assigned to parameters of the base measure, G_0 , (i.e., unknown parameters in the prior for $\tilde{\theta}_j$) ignore cluster parameters associated with clusters that are not occupied, since their current values are meaningless. We assign a special slice sampler that determines the occupied clusters in any given iteration, called slice_CRP_base_param. Note that if you choose to use a different sampler for the base measure parameters, you should also avoid using the CRP_cluster_wrapper sampler.

Finally, a specialized sampler is assigned to the conc hyper parameter when a gamma hyper prior is assigned, see section 6 in Escobar and West (1995) for more details. Otherwise, a random walk Metropolis-Hastings sampler is assigned.

10.4.1.1 Initial values

Valid initial values should be provided for all elements of the process specified by a CRP structure before running the MCMC. A simple and safe choice for \mathbf{z} is to provide a sample of size \mathbb{N} , the same as its length, of values between 1 and some reasonable number of clusters (less than or equal to the length of \mathbf{z}), with replacement, as done in the preceding CRP example. For the concentration parameter, a safe initial value is 1.

10.4.1.2 Sampling the random measure

In BNP models, it is oftenly of interest to make inference about the unknown measure G. NIMBLE provides the getSamplesDPmeasure sampler to generate posterior samples of the random measure G when a CRP structure is involved in the model.

The getSamplesDPmeasure function has two arguments: MCMC and epsilon.

The MCMC argument is a compiled or uncompiled MCMC object. The MCMC object should monitor the membership (or clustering) variable, the cluster parameters, all stochastic nodes of the cluster parameters, and the concentration parameter, if it is random. Use the monitors argument when configuring the MCMC to ensure these variables are monitored.

The epsilon argument is used to determine the truncation level of the representation of the random measure. Its default value is 1e - 04.

The sampler is used only after the MCMC for the model has been run; more details are available from help(getsamplesDPmeasure).

Posterior samples of the random measure G are (almost surely) of the form $\sum_{l=1}^{\infty} w_l \delta_{\theta_l}$, where δ_{θ} is the Dirac measure at θ , w_l are stick-breaking weights, and θ_l are atoms (or point masses). The variables that define the stick-breaking weights are iid $Beta(1, \alpha + N)$ distributed, where α is the concentration parameter of the CRP distribution and N is the sample size. Independently, the atoms, θ_l , are iid and follow a distribution of the form

$$(\alpha+N)^{-1}\sum_{k=1}^{N^\star(z)}m_k(z)\delta_{\{\tilde{\theta}_k\}}+\alpha(\alpha+N)^{-1}G_0,$$

where G_0 is the prior baseline distribution, and z and $\tilde{\theta}_k$ are posterior samples of the labeling vector and cluster parameters, respectively. Their values, together with the values of α (if random) are obtained from the MCMC's output. Expressions $m_k(z)$ and $N^*(z)$ are defined as in section 10.2.

The getsamplesDPmeasure function provides samples of a truncated version of the infinite mixture to a level L. The truncation level L is such that the tail probability left from the approximation is at most epsilon, denoted ϵ . The following relationship determines the truncation level: $L = \log(\epsilon)/\log[(\alpha+N)/(\alpha+N+1)]$. The value of L varies at each iteration of the MCMC's output when α is random, while it is the same at each iteration when α is fixed. For more details about sampling the random measure and its truncation level see Gelfand and Kottas (2002).

Because of the discrete nature of the atom's distribution, the stick-breaking representation truncated to a level L will generally have repeated values. Therefore, in order to reduce the output's dimensionality, the stick-breaking weights of identical atoms are added up. This results in samples of G that have only the 'unique' atoms in $\{\theta_I\}_{I=1}^L$.

The following code exemplifies how to generate samples from G after defining the model as in Section 10.2.2.

```
cmodel <- compileNimble(model)

monitors <- c('z', 'thetatilde', 's2tilde' , 'alpha')
modelConf <- configureMCMC(model, monitors = monitors)
modelMCMC <- buildMCMC(modelConf)

cmodelMCMC <- compileNimble(modelMCMC, project = model)
cmodelMCMC$run(1000)

samplesG <- getSamplesDPmeasure(cmodelMCMC)</pre>
```

10.4.2 Sampling stick-breaking models

NIMBLE's MCMC engine provides specialized samplers for the beta-distributed random variables that are the arguments to the stick-breaking function, updating each component of the weight vector sequentially. The sampler is assigned based on inspection of the model structure. Specifically, the specialized sampler is assigned when the membership vector has a categorical distribution, its weights are defined by a stick-breaking function, and the vector defining the weights follows a beta distribution.

10.4.2.1 Initial values

Valid initial values should be provided for all elements of the stick-breaking function and membership variable before running the MCMC. A simple and safe choice for z is to provide a sample of size N, of values between 1 and some value less than L, with replacement, as done in the preceding stick-breaking example. For the stick variables, safe initial values can be simulated from a beta distribution.

$\begin{array}{c} {\rm Part~IV} \\ {\rm Programming~with~NIMBLE} \end{array}$

Overview

Part IV is the programmer's guide to NIMBLE. At the heart of programming in NIMBLE are nimbleFunctions. These support two principal features: (1) a *setup* function that is run once for each model, nodes, or other setup arguments, and (2) *run* functions that will be compiled to C++ and are written in a subset of R enhanced with features to operate models. Formally, what can be compiled comprises the NIMBLE language, which is designed to be R-like.

This part of the manual is organized as follows:

- Chapter 11 describes how to write simple nimbleFunctions, which have no setup code and hence don't interact with models, to compile parts of R for fast calculations. This covers the subset of R that is compilable, how to declare argument types and return types, and other information.
- Chapter 12 explains how to write nimbleFunctions that can be included in BUGS code as user-defined distributions or user-defined functions.
- Chapter 13 introduces more features of NIMBLE models that are useful for writing nimble-Functions to use models, focusing on how to query model structure and carry out model calculations.
- Chapter 14 introduces two kinds of data structures: modelValues are used for holding multiple sets of values of model variables; nimbleList data structures are similar to R lists but require fixed element names and types, allowing the NIMBLE compiler to use them.
- Chapter 15 draws on the previous chapters to show how to write nimbleFunctions that work with models, or more generally that have a setup function for any purpose. Typically a setup function queries model structure (Chapter 13) and may establish some modelValues or nimbleList data structures or configurations (Chapter 14). Then run functions written in the same way as simple nimbleFunctions (Chapter 11) along with model operations (Chapter 13) define algorithm computations that can be compiled via C++.

Chapter 11

Writing simple nimbleFunctions

11.1 Introduction to simple nimbleFunctions

nimbleFunctions are the heart of programming in NIMBLE. In this chapter, we introduce simple nimbleFunctions that contain only one function to be executed, in either compiled or uncompiled form, but no setup function or additional methods.

Defining a simple nimbleFunction is like defining an R function: nimbleFunction returns a function that can be executed, and it can also be compiled. Simple nimbleFunctions are useful for doing math or the other kinds of processing available in NIMBLE when no model or modelValues is needed. These can be used for any purpose in R programming. They can also be used as new functions and distributions in NIMBLE's extension of BUGS (Chapter 12).

Here's a basic example implementing the textbook calculation of least squares estimation of linear regression parameters¹:

```
solveLeastSquares <- nimbleFunction(
   run = function(X = double(2), y = double(1)) { # type declarations
        ans <- inverse(t(X) %*% X) %*% (t(X) %*% y)
        return(ans)
        returnType(double(2)) # return type declaration
   })

X <- matrix(rnorm(400), nrow = 100)
y <- rnorm(100)
solveLeastSquares(X, y)</pre>
```

```
## [,1]
## [1,] 0.088547575
## [2,] 0.002025635
## [3,] -0.090387283
## [4,] -0.072660391
```

¹Of course, in general, explicitly calculating the inverse is not the recommended numerical recipe for least squares.

```
CsolveLeastSquares <- compileNimble(solveLeastSquares)
CsolveLeastSquares(X, y)</pre>
```

```
## [,1]
## [1,] 0.088547575
## [2,] 0.002025635
## [3,] -0.090387283
## [4,] -0.072660391
```

In this example, we fit a linear model for 100 random response values (y) to four columns of randomly generated explanatory variables (X). We ran the nimbleFunction solveLeastSquares uncompiled, natively in R, allowing testing and debugging (Section 15.7). Then we compiled it and showed that the compiled version does the same thing, but faster². NIMBLE's compiler creates C++ that uses the Eigen (http://eigen.tuxfamily.org) library for linear algebra.

Notice that the actual NIMBLE code is written as an R function definition that is passed to nimbleFunction as the run argument. Hence we call it the *run* code. run code is written in the NIMBLE language. This is similar to a narrow subset of R with some additional features. Formally, we view it as a distinct language that encompasses what can be compiled from a nimbleFunction.

To write nimbleFunctions, you will need to learn:

- what R functions are supported for NIMBLE compilation and any ways they differ from their regular R counterparts;
- how NIMBLE handles types of variables;
- how to declare types of nimbleFunction arguments and return values;
- that compiled nimbleFunctions always pass arguments to each other by reference.

The next sections cover each of these topics in turn.

11.2 R functions (or variants) implemented in NIMBLE

11.2.1 Finding help for NIMBLE's versions of R functions

Often, R help pages are available for NIMBLE's versions of R functions using the prefix 'nim' and capitalizing the next letter. For example, help on NIMBLE's version of numeric can be found by help(nimNumeric). In some cases help is found directly using the name of the function as it appears in R.

11.2.2 Basic operations

Basic R operations supported for NIMBLE compilation are listed in Table 11.1.

²On the machine this is being written on, the compiled version runs a few times faster than the uncompiled version. However we refrain from formal speed tests.

Table 11.1: Basic R manipulation functions in NIMBLE. For some of these functions, in addition to the usual use of help(), you can find help in R for NIMBLE's version of a function by using the "nim" prefix and capitalizing the next letter. E.g. help(nimC) for help with c().

Function	Comments (differences from R)
c()	No recursive argument.
rep()	No rep.int or rep_len arguments.
seq() and ':'	Negative integer sequences from ':', e.g., 2:1 do not work.
which()	No arr.ind or useNames arguments.
diag()	Works like R in three ways: diag(vector) returns a matrix with vector on the diagonal;
	diag(matrix) returns the diagonal vector of matrix;
	diag(n) returns an $n \times n$ identity matrix. No nrow or ncol arguments.
diag()<-	Works for assigning the diagonal vector of a matrix.
<pre>dim()</pre>	Works on a vector as well as higher-dimensional arguments.
length()	
is.na()	Does not correctly handle NAs from R that are type 'logical',
	so convert these using as.numeric() before passing from R to NIMBLE.
<pre>is.nan()</pre>	Treats NA of type 'double' as being NaN and NA of type 'logical' as not being NaN.
any()	One argument only; NAs treated as FALSE.
all()	One argument only; NAs treated as FALSE.
<pre>numeric()</pre>	Allows additional arguments to control initialization.
logical()	Allows additional arguments to control initialization.
<pre>integer()</pre>	Allows additional arguments to control initialization.
<pre>matrix()</pre>	Allows additional arguments to control initialization.
array()	Allows additional arguments to control initialization.
indexing	Arbitrary integer and logical indexing is supported for objects of one or two dimensions.
	For higher-dimensional objects, only: indexing works and then only to create an object
	of at most two dimensions.

In addition to the above functions, we provide functions any_na() and any_nan() for finding if there are any NA or NaN values in a vector. These are equivalent to using any(is.na()) and any(is.nan()) in R.

Other R functions with numeric arguments and return value can be called during compiled execution by wrapping them as a nimbleRcall (see Section 11.7).

Next we cover some of these functions in more detail.

11.2.2.1 numeric, integer, logical, matrix and array

numeric, integer, or logical will create a 1-dimensional vector of floating-point (or 'double' [precision]), integer, or logical values, respectively. The length argument specifies the vector length (default 0), and the value argument specifies the initial value either as a scalar (used for all vector elements, with default 0) or a vector. If a vector and its length is not equal to length, the remaining values will be zero, but we plan to implement R-style recycling in the next version of NIMBLE. The init argument specifies whether or not to initialize the elements in compiled code (default TRUE).

If first use of the variable does not rely on initial values, using init = FALSE will yield slightly more efficient performance.

matrix creates a 2-dimensional matrix object of either floating-point (if type = "double", the default), integer (if type = "integer"), or logical (if type = "logical") values. As in R, nrow and ncol arguments specify the number of rows and columns, respectively. The value and init arguments are used in the same way as for numeric, integer, and logical.

array creates a vector or higher-dimensional object, depending on the dim argument, which takes a vector of sizes for each dimension. The type, value and init argument behave the same as for matrix.

The best way to create an identity matrix is with diag(n), which returns an $n \times n$ identity matrix. NIMBLE also provides a deprecated nimbleFunction identityMatrix that does the same thing.

Examples of these functions, and the related function 'setSize' for changing the size of a numeric object, are given in Section 11.3.2.4.

11.2.2.2 length and dim

length behaves like R's length function. It returns the *entire* length of X. That means if X is multivariate, length(X) returns the product of the sizes of the dimensions. NIMBLE's version of dim, which has synonym nimDim, behaves like R's dim function for matrices or arrays and like R's length function for vectors. In other words, regardless of whether the number of dimensions is 1 or more, it returns a vector of the sizes.

11.2.2.3 Deprecated creation of non-scalar objects using declare

Previous versions of NIMBLE provided a function declare for declaring variables. The more R-like functions numeric, integer, logical, matrix and array are intended to replace declare, but declare is still supported for backward compatibility. In a future version of NIMBLE, declare may be removed.

11.2.3 Math and linear algebra

Numeric scalar and matrix mathematical operations are listed in Tables 11.2-11.3.

As in R, many scalar operations in NIMBLE will work component-wise on vectors or higher dimensional objects. For example if B and C are vectors, A = B + C will add them and create vector C by component-wise addition of B and C. In the current version of NIMBLE, component-wise operations generally only work for vectors and matrices, not arrays with more than two dimensions. The only exception is assignment: A = B will work up to NIMBLE's current limit of four dimensions.

Table 11.2: Functions operating on scalars, many of which can operate on each element (component-wise) of vectors and matrices. Status column indicates if the function is currently provided in NIMBLE. Vector input column indicates if the function can take a vector as an argument (i.e., if the function is vectorized).

$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	Usage	Description	Comments	Status	Vector input
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	x y, x & y	logical OR () and AND(&)		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$! x	logical not		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	x > y, $x >= y$	greater than (and or equal to)		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$x < y$, $x \le y$	less than (and or equal to)		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	x != y, x == y	(not) equals		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	x + y, x - y, x * y	component-wise operators	mix of scalar and vector	yes	yes
$ \begin{array}{cccccccccccccccccccccccccccccccccccc$		component-wise division	vector x and scalar y	yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	x^y, pow(x, y)	power	x^y ; vector x,scalar y	yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	x %% y	modulo (remainder)		yes	no
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	min(x1, x2),	min. (max.) of two scalars		yes	See pmin,
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	max(x1, x2)				pmax
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	exp(x)	exponential		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	_	natural logarithm			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	_	square root			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	_	absolute value		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	step(x)	step function at 0	0 if $x < 0, 1$ if $x >= 0$		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	_	equality of two scalars	1 if $x == y$, 0 if $x! = y$		
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	cube(x)	third power			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	sin(x), cos(x),	trigonometric functions		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	tan(x)				
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	asin(x), acos(x),	inverse trigonometric functions		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	atan(x)				
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	asinh(x), acosh(x),	inv. hyperbolic trig. functions		yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	atanh(x)				
$\begin{array}{llllllllllllllllllllllllllllllllllll$	logit(x)	logit	$\log(x/(1-x))$	yes	yes
$\begin{array}{llllllllllllllllllllllllllllllllllll$	<pre>ilogit(x), expit(x)</pre>	inverse logit	$\exp(x)/(1+\exp(x))$	yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	<pre>probit(x)</pre>	probit (Gaussian quantile)	$\Phi^{-1}(x)$	yes	yes
icloglog(x) inverse complementary log log $1-\exp(-\exp(x))$ yes yes ceiling(x) ceiling function $[(x)]$ yes yes floor(x) floor function $[(x)]$ yes yes round(x) round to integer yes yes trunc(x) truncation to integer yes yes lgamma(x), loggam(x) log gamma function $\log \Gamma(x)$ yes yes besselK(k, nu, modified bessel function returns vector even if yes yesexpon.scaled) of the second kind k a matrix/array k log for k a matrix/array k log for k and k a matrix/array k log for k yes yes	<pre>iprobit(x), phi(x)</pre>	inverse probit (Gaussian CDF)	$\Phi(x)$	yes	yes
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	cloglog(x)	complementary log log	$\log(-\log(1-x))$	yes	yes
floor(x) floor function $[(x)]$ yes yes round(x) round to integer yes yes trunc(x) truncation to integer yes yes lgamma(x), loggam(x) log gamma function $\log \Gamma(x)$ yes yes besselK(k, nu, modified bessel function returns vector even if yes yesexpon.scaled) of the second kind k a matrix/array log1p(x) log of $1+x$ log $(1+x)$ yes yes	icloglog(x)	inverse complementary log log	$1 - \exp(-\exp(x))$	yes	yes
round(x) round to integer yes yes trunc(x) truncation to integer yes yes lgamma(x), loggam(x) log gamma function $\log \Gamma(x)$ yes yes besselK(k, nu, modified bessel function returns vector even if yes yesexpon.scaled) of the second kind k a matrix/array log1p(x) log of $1+x$ log($1+x$) yes yes	<pre>ceiling(x)</pre>	ceiling function	$\lceil (x) \rceil$	yes	yes
trunc(x) truncation to integer yes yes lgamma(x), loggam(x) log gamma function $\log \Gamma(x)$ yes yes besselK(k, nu, modified bessel function returns vector even if yes yesexpon.scaled) of the second kind k a matrix/array $\log 1p(x)$ log of $1+x$ log $(1+x)$ yes yes	floor(x)	floor function	$\lfloor (x) \rfloor$	yes	yes
$\begin{array}{llllllllllllllllllllllllllllllllllll$	round(x)	round to integer		yes	yes
besselK(k, nu, modified bessel function returns vector even if yes yesexpon.scaled) of the second kind k a matrix/array $\log 1p(x)$ $\log of 1 + x$ $\log (1+x)$ yes yes	trunc(x)	truncation to integer		yes	yes
expon.scaled) of the second kind k a matrix/array $\log 1p(x)$ $\log 61 + x$ $\log (1+x)$ yes yes	<pre>lgamma(x), loggam(x)</pre>	log gamma function	$\log \Gamma(x)$	yes	yes
$\log \operatorname{log}(x)$ $\log \operatorname{of} 1 + x$ $\log(1+x)$ yes yes	besselK(k, nu,	modified bessel function	returns vector even if	yes	yes
	expon.scaled)	of the second kind	k a matrix/array		
	log1p(x)	$\log \text{ of } 1 + x$	$\log(1+x)$	yes	yes
	<pre>lfactorial(x),</pre>	log factorial	$\log x!$	yes	yes

Usage	Description	Comments	Status	Vector input
logfact(x) qDIST(x, PARAMS) pDIST(x, PARAMS) rDIST(x, PARAMS) dDIST(x, PARAMS) sort(x) rank(x, s) ranked(x, s)	"q" distribution functions "p" distribution functions "r" distribution functions "d" distribution functions	canonical parameterization canonical parameterization canonical parameterization canonical parameterization	yes	yes yes yes yes
order(x)			no	

Table 11.3: Functions operating on vectors and matrices. Status column indicates if the function is currently provided in NIMBLE.

Usage	Description	Comments	Status
inverse(x)	matrix inverse	x symmetric, positive def.	yes
chol(x)	matrix Cholesky factorization	x symmetric, positive def.	yes
		returns upper triang. matrix	
t(x)	matrix transpose	$x^{ op}$	yes
x%*%y	matrix multiply	xy; x, y conformant	yes
<pre>inprod(x, y)</pre>	dot product	$x^{\top}y$; x and y vectors	yes
solve(x,y)	solve system of equations	$x^{-1}y$; y matrix or vector	yes
forwardsolve(x, y)	solve lower-triangular system of equations	$x^{-1}y$; x lower-triangular	yes
backsolve(x, y)	solve upper-triangular system of equations	$x^{-1}y$; x upper-triangular	yes
logdet(x)	log matrix determinant	$\log x $	yes
asRow(x)	convert vector to 1-row matrix	sometimes automatic	yes
asCol(x)	convert vector to 1-column matrix	sometimes automatic	yes
sum(x)	sum of elements of x		yes
mean(x)	mean of elements of x		yes
sd(x)	standard deviation of elements of x		yes
<pre>prod(x)</pre>	product of elements of x		yes
min(x), max(x)	min. (max.) of elements of x		yes
<pre>pmin(x, y), pmax(x,y)</pre>	vector of mins (maxs) of elements of		yes
	x and y		
<pre>interp.lin(x, v1, v2)</pre>	linear interpolation		no
eigen(x)	matrix eigendecomposition; returns a	x symmetric	yes
	nimbleList of type eigenNimbleList		
svd(x)	matrix singular value decomposition;		yes
	returns a nimbleList of type		
	svdNimbleList		

More information on the nimbleLists returned by the eigen and svd functions in NIMBLE can be found in Section 14.2.1.

11.2.4 Distribution functions

Distribution 'd', 'r', 'p', and 'q' functions can all be used from nimbleFunctions (and in BUGS model code), but care is needed in the syntax, as follows.

- Names of the distributions generally (but not always) match those of R, which sometimes differ from BUGS. See the list below.
- Supported parameterizations are also indicated in the list below.
- For multivariate distributions (multivariate normal, Dirichlet, and Wishart), 'r' functions only return one random draw at a time, and the first argument must always be 1.
- R's recycling rule (re-use of an argument as needed to accommodate longer values of other arguments) is generally followed, but the returned object is always a scalar or a vector, not a matrix or array.

As in R (and nimbleFunctions), arguments are matched by order or by name (if given). Standard arguments to distribution functions in R (log, log.p, lower.tail) can be used and have the same defaults. User-defined distributions for BUGS (Chapter 12) can also be used from nimbleFunctions.

For standard distributions, we rely on R's regular help pages (e.g., help(dgamma). For distributions unique to NIMBLE (e.g., dexp_nimble, ddirch), we provide help pages.

Supported distributions, listed by their 'd' function, include:

- dbinom(x, size, prob, log)
- dcat(x, prob, log)
- dmulti(x, size, prob, log)
- dnbinom(x, size, prob, log)
- dpois(x, lambda, log)
- dbeta(x, shape1, shape2, log)
- dchisq(x, df, log)
- ddexp(x, location, rate, log)
- ddexp(x, location, scale, log)
- dexp(x, rate, log)
- dexp nimble(x, rate, log)
- dexp_nimble(x, scale, log)
- dgamma(x, shape, rate, log)
- dgamma(x, shape, scale, log)
- dinvgamma(x, shape, rate, log)
- dinvgamma(x, shape, scale, log)
- dlnorm(x, meanlog, sdlog, log)
- dlogis(x, location, scale, log)
- dnorm(x, mean, sd, log)
- dt_nonstandard(x, df, mu, sigma, log)
- dt(x, df, log)
- dunif(x, min, max, log)
- dweibull(x, shape, scale, log)
- ddirch(x, alpha, log)
- dlkj_corr_cholesky(x, shape, size, log)
- dmnorm_chol(x, mean, cholesky, prec_param, log)

- dmvt_chol(x, mu, cholesky, df, prec_param, log)
- dwish_chol(x, cholesky, df, scale_param, log)
- dinvwish_chol(x, cholesky, df, scale_param, log)

In the last three, cholesky stands for Cholesky decomposition of the relevant matrix; prec_param is a logical indicating whether the Cholesky is of a precision matrix (TRUE) or covariance matrix (FALSE)³; and scale_param is a logical indicating whether the Cholesky is of a scale matrix (TRUE) or an inverse scale matrix (FALSE).

11.2.5 Flow control: if-then-else, for, while, and stop

These basic flow-control structures use the same syntax as in R. However, for-loops are limited to sequential integer indexing. For example, for(i in 2:5) {...} works as it does in R. Decreasing index sequences are not allowed. Unlike in R, if is not itself a function that returns a value.

We plan to include more flexible for-loops in the future, but for now we've included just one additional useful feature: for(i in seq_along(NFL)) will work as in R, where NFL is a nimbleFunctionList. This is described in Section 15.4.8.

stop, or equivalently nimStop, throws control to R's error-handling system and can take a character argument that will be displayed in an error message.

11.2.6 print and cat

print, or equivalently nimPrint, prints an arbitrary set of outputs in order and adds a newline character at the end. cat or nimCat is identical, except without a newline at the end.

11.2.7 Checking for user interrupts: checkInterrupt

When you write algorithms that will run for a long time in C++, you may want to explicitly check whether a user has tried to interrupt the execution (i.e., by pressing Control-C). Simply include checkInterrupt in run code at places where a check should be done. If there has been an interrupt waiting to be handled, the process will stop and return control to R.

11.2.8 Optimization: optim and nimOptim

NIMBLE provides a nimOptim function that partially implement's R's optim function with some minor differences. nimOptim supports optimization methods 'Nelder-Mead', 'BFGS', 'CG', 'L-BFGS-B', but does not support methods 'SANN' and 'Brent'. NIMBLE's nimOptim supports gradients using user-provided functions if available or finite differences otherwise, but it does not currently support Hessian computations. Currently nimOptim does not support extra parameters to the function being optimized (via \dots), but a work-around is to create a new nimbleFunction that binds those fixed parameters. Finally, nimOptim requires a nimbleList datatype for the control parameter, whereas R's optim uses a simple R list. To define the control parameter, create a default value with the nimOptimDefaultControl function, and set any desired fields. For example usage, see the unit tests in tests/test-optim.R.

³For the multivariate t, these are more properly termed the 'inverse scale' and 'scale' matrices

11.2.9 'nim' synonyms for some functions

NIMBLE uses some keywords, such as dim and print, in ways similar but not identical to R. In addition, there are some keywords in NIMBLE that have the same names as R functions with quite different functionality. For example, step is part of the BUGS language, but it is also an R function for stepwise model selection. And equals is part of the BUGS language but is also used in the testthat package, which we use in testing NIMBLE.

NIMBLE tries to avoid conflicts by replacing some keywords immediately upon creating a nimble-Function. These replacements include

- $c \rightarrow \text{nim} C$
- $copy \rightarrow nimCopy$
- $\bullet \ \ \mathtt{dim} \to \mathtt{nim}\mathtt{Dim}$
- print \rightarrow nimPrint
- $\bullet \ \mathtt{cat} \to \mathtt{nimCat}$
- $step \rightarrow nimStep$
- equals \rightarrow nimEquals
- $rep \rightarrow nimRep$
- round \rightarrow nimRound
- $\operatorname{\mathsf{seq}} \to \operatorname{\mathsf{nimSeq}}$
- $\mathsf{stop} \to \mathsf{nimStop}$
- ullet switch ightarrow nimSwitch
- numeric, integer, logical → nimNumeric, nimInteger, nimLogical
- matrix, array → nimMatrix, nimArray

This system gives programmers the choice between using the keywords like nimPrint directly, to avoid confusion in their own code about which 'print' is being used, or to use the more intuitive keywords like print but remember that they are not the same as R's functions.

11.3 How NIMBLE handles types of variables

Variables in the NIMBLE language are statically typed. Once a variable is used for one type, it can't subsequently be used for a different type. This rule facilitates NIMBLE's compilation to C++. The NIMBLE compiler often determines types automatically, but sometimes the programmer needs to explicitly provide them.

The elemental types supported by NIMBLE include double (floating-point), integer, logical, and character. The type of a numeric or logical object refers to the number of dimensions and the elemental type of the elements. Hence if x is created as a double matrix, it can only be used subsequently for a double matrix. The size of each dimension is not part of its type and thus can be changed. Up to four dimensions are supported for double, integer, and logical. Only vectors (one dimension) are supported for character. Unlike R, NIMBLE supports true scalars, which have 0 dimensions.

11.3.1 nimbleList data structures

A nimbleList is a data structure that can contain arbitrary other NIMBLE objects, including other nimbleLists. Like other NIMBLE types, nimbleLists are strongly typed: each nimbleList

is created from a configuration that declares what types of objects it will hold. nimbleLists are covered in Chapter 14.2.

11.3.2 How numeric types work

R's dynamic types support easy programming because one type can sometimes be transformed to another type automatically when an expression is evaluated. NIMBLE's static types makes it stricter than R.

11.3.2.1 When NIMBLE can automatically set a numeric type

When a variable is first created by assignment, its type is determined automatically by that assignment. For example, if x has not appeared before, then

```
x \leftarrow A \% \% B \# assume A and B are double matrices or vectors
```

will create x to be a double matrix of the correct size (determined during execution).

11.3.2.1.1 Avoid changing types of a variable within a nimbleFunction Because NIMBLE is statically typed, you cannot use the same variable name for two objects of different types (including objects of different dimensions).

Suppose we have (implicitly) created x as a double matrix. If x is used subsequently, it can only be used as a double matrix. This is true even if it is assigned a new value, which will again set its size automatically but cannot change its type.

```
x <- A %*% B # assume A and B are double matrices or vectors
x <- nimMatrix(0, nrow = 5, ncol = 2) # OK: 'x' is still a double matrix
x <- rnorm(10) # NOT OK: 'x' is a double vector</pre>
```

11.3.2.2 When a numeric object needs to be created before being used

If the contents of a variable are to be populated by assignment into some indices in steps, the variable must be created first. Further, it must be large enough for its eventual contents; it will not be automatically resized if assignments are made beyond its current size. For example, in the following code, \mathbf{x} must be created before being filled with contents for specific indices.

```
x <- numeric(10)

for(i in 1:10)
    x[i] <- foo(y[i])</pre>
```

11.3.2.3 How NIMBLE handles NA

NIMBLE supports use of NA with some caveats. In the compiled version of a nimbleFunction:

- NA values in a logical scalar or vector will not work;
- Assigning NA to a new variable will make that variable have type double (in R, this would be a logical!);
- Non-trivial use of NA in integer variables may fail by having the value become a large (negative) number;

• Use of NA in doubles should generally work.

These issues arise because NIMBLE uses R's encoding of NA values in C(++) but uses native compiled math and type-casting in C++, which do not always preserve R's NA encodings. In summary, NA should generally work for doubles and should be used cautiously (i.e. after you test that what you need to work actually works) for integers. For some needs, NaN is a suitable alternative to NA.

11.3.2.4 Changing the sizes of existing objects: setSize

setSize changes the size of an object, preserving its contents in column-major order.

```
# Example of creating and resizing a floating-point vector
# myNumericVector will be of length 10, with all elements initialized to 2
myNumericVector <- numeric(10, value = 2)
# resize this numeric vector to be length 20; last 10 elements will be 0
setSize(myNumericVector, 20)

# Example of creating a 1-by-10 matrix with values 1:10 and resizing it
myMatrix <- matrix(1:10, nrow = 1, ncol = 10)
# resize this matrix to be a 10-by-10 matrix
setSize(myMatrix, c(10, 10))
# The first column will have the 1:10</pre>
```

11.3.2.5 Confusions between scalars and length-one vectors

In R, there is no such thing is a true scalar; scalars can always be treated as vectors of length one. NIMBLE allows true scalars, which can create confusions. For example, consider the following code:

```
myfun <- nimbleFunction(
    run = function(i = integer()) { # i is an integer scalar
        randomValues <- rnorm(10)  # double vector
        a <- randomValues[i]  # double scalar
        b <- randomValues[i:i]  # double vector
        d <- a + b  # double vector
        f <- c(i)  # integer vector
})</pre>
```

In the line that creates b, the index range i:i is not evaluated until run time. Even though i:i will always evaluate to simpy i, the compiler does not determine that. Since there is a vector index range provided, the result of randomValues[i:i] is determined to be a vector. The following line then creates d as a vector, because a vector plus a scalar returns a vector. Another way to create a vector from a scalar is to use c, as illustrated in the last line.

11.3.2.6 Confusions between vectors and one-column or one-row matrices

Consider the following code:

```
myfun <- nimbleFunction(
    run = function() {</pre>
```

```
A <- matrix(value = rnorm(9), nrow = 3)

B <- rnorm(3)

Cmatrix <- A %*% B  # double matrix, one column

Cvector <- (A %*% B)[,1]  # double vector

Cmatrix <- (A %*% B)[,1]  # error, vector assigned to matrix

Cmatrix[,1] <- (A %*% B)[,1]  # ok, if Cmatrix is large enough

})
```

This creates a matrix A, a vector B, and matrix-multiplies them. The vector B is automatically treated as a one-column matrix in matrix algebra computations. The result of matrix multiplication is always a matrix, but a programmer may expect a vector, since they know the result will have one column. To make it a vector, simply extract the first column. More information about such handling is provided in the next section.

11.3.2.7 Understanding dimensions and sizes from linear algebra

As much as possible, NIMBLE behaves like R when determining types and sizes returned from linear algebra expressions, but in some cases this is not possible because R uses run-time information while NIMBLE must determine dimensions at compile time. For example, when matrix multiplying a matrix by a vector, R treats the vector as a one-column matrix unless treating it as a one-row matrix is the only way to make the expression valid, as determined at run time. NIMBLE usually must assume during compilation that it should be a one-column matrix, unless it can determine not just the number of dimensions but the actual sizes during compilation. When needed asRow and asCol can control how a vector will be treated as a matrix.

Here is a guide to such issues. Suppose v1 and v2 are vectors, and M1 is a matrix. Then

- v1 + M1 generates a compilation error unless one dimension of M1 is known at compile-time to be 1. If so, then v1 is promoted to a 1-row or 1-column matrix to conform with M1, and the result is a matrix of the same sizes. This behavior occurs for all component-wise binary functions.
- v1 %*% M1 defaults to promoting v1 to a 1-row matrix, unless it is known at compile-time that M1 has 1 row, in which case v1 is promoted to a 1-column matrix.
- M1 %*% v1 defaults to promoting v1 to a 1-column matrix, unless it is known at compile time that M1 has 1 column, in which case v1 is promoted to a 1-row matrix.
- v1 %*% v2 promotes v1 to a 1-row matrix and v2 to a 1-column matrix, so the returned values is a 1x1 matrix with the inner product of v1 and v2. If you want the inner product as a scalar, use inprod(v1, v2).
- asRow(v1) explicitly promotes v1 to a 1-row matrix. Therefore v1 %*% asRow(v2) gives the outer product of v1 and v2.
- asCol(v1) explicitly promotes v1 to a 1-column matrix.
- The default promotion for a vector is to a 1-column matrix. Therefore, v1 % t(v2) is equivalent to v1 % asRow(v2).
- When indexing, dimensions with scalar indices will be dropped. For example, M1[1,] and M1[,1] are both vectors. If you do not want this behavior, use drop=FALSE just as in R. For example, M1[1,,drop=FALSE] is a matrix.
- The left-hand side of an assignment can use indexing, but if so it must already be correctly sized for the result. For example, Y[5:10, 20:30] <- x will not work and could crash

your R session with a segmentation fault – if Y is not already at least 10x30 in size. This can be done by setSize(Y, c(10, 30)). See Section 11.3.2.4 for more details. Note that non-indexed assignment to Y, such as Y <- x, will automatically set Y to the necessary size.

Here are some examples to illustrate the above points, assuming M2 is a square matrix.

- Y <- v1 + M2 %*% v2 will return a 1-column matrix. If Y is created by this statement, it will be a 2-dimensional variable. If Y already exists, it must already be 2-dimensional, and it will be automatically re-sized for the result.
- Y <- v1 + (M2 %*% v2)[,1] will return a vector. Y will either be created as a vector or must already exist as a vector and will be re-sized for the result.

11.3.2.8 Size warnings and the potential for crashes

For matrix algebra, NIMBLE cannot ensure perfect behavior because sizes are not known until run time. Therefore, it is possible for you to write code that will crash your R session. In Version 0.13.2, NIMBLE attempts to issue a warning if sizes are not compatible, but it does not halt execution. Therefore, if you execute A <- M1 %*% M2, and M1 and M2 are not compatible for matrix multiplication, NIMBLE will output a warning that the number of rows of M1 does not match the number of columns of M2. After that warning the statement will be executed and may result in a crash. Another easy way to write code that will crash is to do things like Y[5:10, 20:30] <- x without ensuring Y is at least 10x30. In the future we hope to prevent crashes, but in Version 0.13.2 we limit ourselves to trying to provide useful information.

11.4 Declaring argument and return types

NIMBLE requires that types of arguments and the type of the return value be explicitly declared.

As illustrated in the example in Section 11.1, the syntax for a type declaration is:

```
type(nDim, sizes)
```

where type is double, integer, logical or character. (In more general nimbleFunction programming, a type can also be a nimbleList type, discussed in Section 14.2.)

For example run = $function(x = double(1)) \{ ... \}$ sets the single argument of the run function to be a vector of numeric values of unknown size.

For type(nDim, sizes), nDim is the number of dimensions, with 0 indicating scalar and omission of nDim defaulting to a scalar. sizes is an optional vector of fixed, known sizes.

For example, double(2, c(4, 5)) declares a 4×5 matrix. If sizes are omitted, they will be set either by assignment or by setSize.

In the case of scalar arguments only, a default value can be provided. For example, to provide 1.2 as a default:

```
myfun <- nimbleFunction(
    run = function(x = double(0, default = 1.2)) {
})</pre>
```

Functions with return values must have their return type explicitly declared using returnType, which can occur anywhere in the run code. For example returnType(integer(2)) declares the

return type to be a matrix of integers. A return type of void() means there is no return value, which is the default if no returnType statement is included.

Note that because all values in models are stored as doubles and because of some limitations in NIMBLE's automatic casting, non-scalar return values of user-defined distributions must be doubles.

11.5 Compiled nimbleFunctions pass arguments by reference

Uncompiled nimbleFunctions pass arguments like R does, by copy. If x is passed as an argument to function foo, and foo modifies x internally, it is modifying its copy of x, not the original x that was passed to it.

Compiled nimbleFunctions pass arguments to other compiled nimbleFunctions by reference (or pointer). This is very different. Now if foo modifies x internally, it is modifying the same x that was passed to it. This allows much faster execution but is obviously a fundamentally different behavior.

Uncompiled execution of nimbleFunctions is primarily intended for debugging. However, debugging of how nimbleFunctions interact via arguments requires testing the compiled versions.

11.6 Calling external compiled code

If you have a function in your own compiled C or C++ code and an appropriate header file, you can generate a nimbleFunction that wraps access to that function, which can then be used in other nimbleFunctions. See help(nimbleExternalCall) for an example. This also contains an example of using an externally compiled function in the BUGS code of a model.

11.7 Calling uncompiled R functions from compiled nimbleFunctions

Sometimes one may want to combine R functions with compiled nimbleFunctions. Obviously a compiled nimbleFunction can be called from R. An R function with numeric inputs and output can be called from compiled nimbleFunctions. The call to the R function is wrapped in a nimbleFunction returned by nimbleRcall. See help(nimbleRcall) for an example, including an example of using the resulting function in the BUGS code of a model.

Chapter 12

Creating user-defined BUGS distributions and functions

NIMBLE allows you to define your own functions and distributions as *nimbleFunctions* for use in BUGS code. As a result, NIMBLE frees you from being constrained to the functions and distributions discussed in Chapter 5. For example, instead of setting up a Dirichlet prior with multinomial data and needing to use MCMC, one could recognize that this results in a Dirichlet-multinomial distribution for the data and provide that as a user-defined distribution instead.

Since NIMBLE allows you to wrap calls to external compiled code or arbitrary R functions as nimbleFunctions, and since you can define model functions and distributions as nimbleFunctions, you can combine these features to build external compiled code or arbitrary R functions into a model. See Sections 11.6-11.7.

Note that NIMBLE generally expects user-defined distributions or functions to be defined in the global environment. If you define them in a function (which would generally be the case if you are using them in the context of parallelization), one approach would be to assign them to the global environment in your function:

```
## for a user-defined function
assign('myfun', myfun, envir = .GlobalEnv)
## for a user-defined distribution
assign('dfoo', dfoo, envir = .GlobalEnv)
assign('rfoo', rfoo, envir = .GlobalEnv)
## similarly for 'p' and 'q' functions if you define them
```

12.1 User-defined functions

To provide a new function for use in BUGS code, simply create a nimbleFunction that has no *setup* code as discussed in Chapter 11. Then use it in your BUGS code. That's it.

Writing nimbleFunctions requires that you declare the dimensionality of arguments and the returned object (Section 11.4). Make sure that the dimensionality specified in your nimbleFunction matches how you use it in BUGS code. For example, if you define scalar parameters in your BUGS code you will want to define nimbleFunctions that take scalar arguments. Here is an example that returns

twice its input argument:

```
timesTwo <- nimbleFunction(
    run = function(x = double(0)) {
        returnType(double(0))
        return(2*x)
    })

code <- nimbleCode({
    for(i in 1:3) {
        mu[i] ~ dnorm(0, 1)
        mu_times_two[i] <- timesTwo(mu[i])
    }
})</pre>
```

The x = double(0) argument and returnType(double(0)) establish that the input and output will both be zero-dimensional (scalar) numbers.

You can define nimbleFunctions that take inputs and outputs with more dimensions. Here is an example that takes a vector (1-dimensional) as input and returns a vector with twice the input values:

```
vectorTimesTwo <- nimbleFunction(
    run = function(x = double(1)) {
        returnType(double(1))
        return(2*x)
    }
    )
code <- nimbleCode({
    for(i in 1:3) {
        mu[i] ~ dnorm(0, 1)
    }
    mu_times_two[1:3] <- vectorTimesTwo(mu[1:3])
})</pre>
```

There is a subtle difference between the mu_times_two variables in the two examples. In the first example, there are individual nodes for each mu_times_two[i]. In the second example, there is a single multivariate node, mu_times_two[1:3]. Each implementation could be more efficient for different needs. For example, suppose an algorithm modifies the value of mu[2] and then updates nodes that depend on it. In the first example, mu_times_two[2] would be updated. In the second example mu_times_two[1:3] would be updated because it is a single, vector node.

At present in compiled use of a model, you cannot provide a scalar argument where the user-defined nimbleFunction expects a vector; unlike in R, scalars are not simply vectors of length 1.

12.2 User-defined distributions

To provide a user-defined distribution, you need to define density ('d') and (optionally) simulation ('r') nimbleFunctions, without setup code, for your distribution. In many cases you can then simply use your distribution in BUGS code as you would any distribution already provided by

NIMBLE, while in a few special cases¹ you need to explicitly register your distribution as described in Section 12.2.1.

You need to provide the simulation ('r') function if any algorithm used with a model that uses the distribution needs to simulate from the distribution. This is not the case for NIMBLE's built-in MCMC sampler functions, and therefore the simulation function is not generally required for standard MCMC in NIMBLE. However, the 'r' function is necessary for initialization of nodes that are assigned the user-defined distribution if no initial value is provided, and for sampling posterior predictive nodes (those nodes with no downstream data dependencies) that are assigned the user-defined distribution.

You can optionally provide distribution ('p') and quantile ('q') functions, which will allow truncation to be applied to a user-defined distribution. You can also provide a list of alternative parameterizations, but only if you explicitly register the distribution.

Here is an extended example of providing a univariate exponential distribution (solely for illustration as this is already provided by NIMBLE) and a multivariate Dirichlet-multinomial distribution.

```
dmyexp <- nimbleFunction(</pre>
    run = function(x = double(0), rate = double(0, default = 1),
        log = integer(0, default = 0)) {
        returnType(double(0))
        logProb <- log(rate) - x*rate</pre>
        if(log) return(logProb)
        else return(exp(logProb))
    })
rmyexp <- nimbleFunction(</pre>
    run = function(n = integer(0), rate = double(0, default = 1)) {
        returnType(double(0))
        if(n != 1) print("rmyexp only allows n = 1; using n = 1.")
        dev \leftarrow runif(1, 0, 1)
        return(-log(1-dev) / rate)
    })
pmyexp <- nimbleFunction(</pre>
    run = function(q = double(0), rate = double(0, default = 1),
        lower.tail = integer(0, default = 1),
        log.p = integer(0, default = 0)) {
        returnType(double(0))
        if(!lower.tail) {
            logp <- -rate * q
             if(log.p) return(logp)
             else return(exp(logp))
        } else {
            p <- 1 - exp(-rate * q)</pre>
            if(!log.p) return(p)
```

¹These include providing alternative parameterizations, specifying the range of the distribution, or specifying that the distribution is a discrete distribution.

```
else return(log(p))
        }
    })
qmyexp <- nimbleFunction(</pre>
    run = function(p = double(0), rate = double(0, default = 1),
        lower.tail = integer(0, default = 1),
        log.p = integer(0, default = 0)) {
        returnType(double(0))
        if(log.p) p \leftarrow exp(p)
        if(!lower.tail) p <- 1 - p</pre>
        return(-log(1 - p) / rate)
    })
ddirchmulti <- nimbleFunction(</pre>
    run = function(x = double(1), alpha = double(1), size = double(0),
        log = integer(0, default = 0)) {
        returnType(double(0))
        logProb <- lgamma(size+1) - sum(lgamma(x+1)) + lgamma(sum(alpha)) -</pre>
             sum(lgamma(alpha)) + sum(lgamma(alpha + x)) -
            lgamma(sum(alpha) + size)
        if(log) return(logProb)
        else return(exp(logProb))
    })
rdirchmulti <- nimbleFunction(</pre>
    run = function(n = integer(0), alpha = double(1), size = double(0)) {
        returnType(double(1))
        if(n != 1) print("rdirchmulti only allows n = 1; using n = 1.")
        p <- rdirch(1, alpha)</pre>
        return(rmulti(1, size = size, prob = p))
    })
code <- nimbleCode({</pre>
     y[1:K] ~ ddirchmulti(alpha[1:K], n)
     for(i in 1:K) {
         alpha[i] ~ dmyexp(1/3)
      }
     })
model <- nimbleModel(code, constants = list(K = 5, n = 10))</pre>
```

The distribution-related functions should take as input the parameters for a single parameterization, which will be the canonical parameterization that NIMBLE will use.

Here are more details on the requirements for distribution-related nimbleFunctions, which follow R's conventions:

- Your distribution-related functions must have names that begin with 'd', 'r', 'p' and 'q'. The name of the distribution must not be identical to any of the NIMBLE-provided distributions.
- All simulation ('r') functions must take n as their first argument. Note that you may simply have your function only handle n=1 and return an warning for other values of n.
- NIMBLE uses doubles for numerical calculations, so we suggest simply using doubles in general, even for integer-valued parameters or values of random variables. In fact, non-scalars must be declared as doubles.
- All density functions must have as their last argument log and implement return of the log probability density. NIMBLE algorithms typically use only log = 1, but we recommend you implement the log = 0 case for completeness.
- All distribution and quantile functions must have their last two arguments be (in order) lower.tail and log.p. These functions must work for lower.tail = 1 (i.e., TRUE) and log.p = 0 (i.e., FALSE), as these are the inputs we use when working with truncated distributions. It is your choice whether you implement the necessary calculations for other combinations of these inputs, but again we recommend doing so for completeness.
- Define the nimbleFunctions in R's global environment. Don't expect R's standard scoping to work².

12.2.1 Using register Distributions for alternative parameterizations and providing other information

Behind the scenes, NIMBLE uses the function registerDistributions to set up new distributions for use in BUGS code. In some circumstances, you will need to call registerDistributions directly to provide information that NIMBLE can't obtain automatically from the nimbleFunctions you write.

The cases in which you'll need to explicitly call registerDistributions are when you want to do any of the following:

- provide alternative parameterizations,
- indicate a distribution is discrete, and
- provide the range of possible values for a distribution.

If you would like to allow for multiple parameterizations, you can do this via the Rdist element of the list provided to registerDistributions as illustrated below. If you provide CDF ('p') and inverse CDF (quantile, i.e. 'q') functions, be sure to specify pqAvail = TRUE when you call registerDistributions. Here's an example of using registerDistributions to provide an alternative parameterization (scale instead of rate) and to provide the range for the user-defined exponential distribution. We can then use the alternative parameterization in our BUGS code.

```
registerDistributions(list(
    dmyexp = list(
        BUGSdist = "dmyexp(rate, scale)",
        Rdist = "dmyexp(rate = 1/scale)",
        altParams = c("scale = 1/rate", "mean = 1/rate"),
        pqAvail = TRUE,
        range = c(0, Inf)
        )
```

²NIMBLE can't use R's standard scoping because it doesn't work for R reference classes, and nimbleFunctions are implemented as custom-generated reference classes.

```
code <- nimbleCode({
    y[1:K] ~ ddirchmulti(alpha[1:K], n)
    for(i in 1:K) {
        alpha[i] ~ T(dmyexp(scale = 3), 0, 100)
    }
})

model <- nimbleModel(code, constants = list(K = 5, n = 10),
        inits = list(alpha = rep(1, 5)))</pre>
```

There are a few rules for how you specify the information about a distribution that you provide to registerDistributions:

- The function name in the BUGSdist entry in the list provided to registerDistributions will be the name you can use in BUGS code.
- The names of your nimbleFunctions must match the function name in the Rdist entry. If missing, the Rdist entry defaults to be the same as the BUGSdist entry.
- Your distribution-related functions must take as arguments the parameters in default order, starting as the second argument and in the order used in the parameterizations in the Rdist argument to registerDistributions or the BUGSdist argument if there are no alternative parameterizations.
- You must specify a types entry in the list provided to registerDistributions if the distribution is multivariate or if any parameter is non-scalar.

Further details on using registerDistributions can be found via R help on registerDistributions. NIMBLE uses the same list format as registerDistributions to define its distributions. This list can be found in the R/distributions_inputList.R file in the package source code directory or as the R list nimble:::distributionsInputList.

Chapter 13

Working with NIMBLE models

Here we describe how one can get information about NIMBLE models and carry out operations on a model. While all of this functionality can be used from R, its primary use occurs when writing nimbleFunctions (see Chapter 15). Information about node types, distributions, and dimensions can be used to determine algorithm behavior in *setup* code of nimbleFunctions. Information about node or variable values or the parameter and bound values of a node would generally be used for algorithm calculations in *run* code of nimbleFunctions. Similarly, carrying out numerical operations on a model, including setting node or variable values, would generally be done in run code.

13.1 The variables and nodes in a NIMBLE model

Section 6.2 defines what we mean by variables and nodes in a NIMBLE model and discusses how to determine and access the nodes in a model and their dependency relationships. Here we'll review and go into more detail on the topics of determining the nodes and node dependencies in a model.

13.1.1 Determining the nodes in a model

One can determine the variables in a model using getVarNames and the nodes in a model using getNodeNames, with optional arguments allowing you to select only certain types of nodes. We illustrate here with the pump model from Chapter 2.

```
pump$getVarNames()
## [1] "lifted_d1_over_beta" "theta"
                                                      "lambda"
## [4] "x"
                                                      "beta"
                               "alpha"
pump$getNodeNames()
    [1] "alpha"
                                "beta"
                                                       "lifted_d1_over_beta"
##
##
    [4] "theta[1]"
                                "theta[2]"
                                                       "theta[3]"
    [7] "theta[4]"
                                "theta[5]"
                                                       "theta[6]"
## [10] "theta[7]"
                                "theta[8]"
                                                       "theta[9]"
## [13] "theta[10]"
                                "lambda[1]"
                                                       "lambda[2]"
## [16] "lambda[3]"
                                "lambda[4]"
                                                       "lambda[5]"
## [19] "lambda[6]"
                                "lambda[7]"
                                                       "lambda[8]"
```

```
## [22] "lambda[9]"
                               "lambda[10]"
                                                     "x[1]"
## [25] "x[2]"
                               "x[3]"
                                                     "x[4]"
## [28] "x[5]"
                               "x[6]"
                                                     "x[7]"
## [31] "x[8]"
                               "x[9]"
                                                     "x[10]"
pump$getNodeNames(determOnly = TRUE)
    [1] "lifted_d1_over_beta" "lambda[1]"
                                                     "lambda[2]"
##
   [4] "lambda[3]"
                               "lambda[4]"
                                                     "lambda[5]"
## [7] "lambda[6]"
                               "lambda[7]"
                                                     "lambda[8]"
## [10] "lambda[9]"
                               "lambda[10]"
pump$getNodeNames(stochOnly = TRUE)
                                 "theta[1]"
   [1] "alpha"
                    "beta"
                                             "theta[2]"
                                                         "theta[3]"
                                                                      "theta[4]"
   [7] "theta[5]"
                    "theta[6]"
                                "theta[7]"
                                             "theta[8]"
                                                         "theta[9]"
                                                                      "theta[10]"
## [13] "x[1]"
                    "x[2]"
                                 "x[3]"
                                             "x[4]"
                                                         "x[5]"
                                                                      "x[6]"
## [19] "x[7]"
                    "x[8]"
                                 "x[9]"
                                             "x[10]"
pump$getNodeNames(dataOnly = TRUE)
## [1] "x[1]" "x[2]" "x[3]" "x[4]" "x[5]" "x[6]" "x[7]" "x[8]" "x[9]"
## [10] "x[10]"
```

You can see one lifted node (see next section), lifted_d1_over_beta, involved in a reparameterization to NIMBLE's canonical parameterization of the gamma distribution for the theta nodes.

We can determine the set of nodes contained in one or more nodes or variables using expandNodeNames, illustrated here for an example with multivariate nodes. The returnScalarComponents argument also allows us to return all of the scalar elements of multivariate nodes.

```
## [1] "Y[1]" "Y[2]" "Y[3]" "Y[4]"
multiVarModel2$expandNodeNames(c("X", "Y"), returnScalarComponents = TRUE)

## [1] "X[1, 1]" "X[1, 2]" "X[1, 3]" "X[6, 3]" "X[7, 3]" "X[8, 3]"
## [7] "X[9, 3]" "X[10, 3]" "X[1, 4]" "X[1, 5]" "Y[1]" "Y[2]"
```

```
## [13] "Y[3]" "Y[4]"
```

As discussed in Section 6.2.6, you can determine whether a node is flagged as data using isData.

13.1.2 Understanding lifted nodes

In some cases, NIMBLE introduces new nodes into the model that were not specified in the BUGS code for the model, such as the lifted_d1_over_beta node in the introductory example. For this reason, it is important that programs written to adapt to different model structures use NIMBLE's systems for querying the model graph. For example, a call to pump\$getDependencies("beta") will correctly include lifted_d1_over_beta in the results. If one skips this step and assumes the nodes are only those that appear in the BUGS code, one may not get correct results.

It can be helpful to know the situations in which lifted nodes are generated. These include:

- When distribution parameters are expressions, NIMBLE creates a new deterministic node that contains the expression for a given parameter. The node is then a direct descendant of the new deterministic node. This is an optional feature, but it is currently enabled in all cases.
- 2. As discussed in Section 5.2.6, the use of link functions causes new nodes to be introduced. This requires care if you need to initialize values in stochastic declarations with link functions.
- 3. Use of alternative parameterizations of distributions, described in Section 5.2.4 causes new nodes to be introduced. For example when a user provides the precision of a normal distribution as tau, NIMBLE creates a new node sd <- 1/sqrt(tau) and uses sd as a parameter in the normal distribution. If many nodes use the same tau, only one new sd node will be created, so the computation 1/sqrt(tau) will not be repeated redundantly.

13.1.3 Determining dependencies in a model

Next we'll see how to determine the node dependencies (or 'descendants' or child nodes) in a model. There are a variety of arguments to getDependencies that allow one to specify whether to include the node itself, whether to include deterministic or stochastic or data dependents, etc. By default getDependencies returns descendants up to the next stochastic node on all edges emanating from the node(s) specified as input. This is what would be needed to calculate a Metropolis-Hastings acceptance probability in MCMC, for example.

```
pump$getDependencies("alpha")
    [1] "alpha"
                    "theta[1]"
                                "theta[2]"
                                             "theta[3]"
                                                          "theta[4]"
    [7] "theta[6]" "theta[7]" "theta[8]"
                                             "theta[9]"
                                                          "theta[10]"
pump$getDependencies(c("alpha", "beta"))
##
    [1] "alpha"
                               "beta"
                                                      "lifted_d1_over_beta"
    [4] "theta[1]"
                               "theta[2]"
                                                      "theta[3]"
    [7] "theta[4]"
                               "theta[5]"
                                                      "theta[6]"
                               "theta[8]"
## [10] "theta[7]"
                                                      "theta[9]"
  [13] "theta[10]"
pump$getDependencies("theta[1:3]", self = FALSE)
## [1] "lambda[1]" "lambda[2]" "lambda[3]" "x[1]"
                                                         "x[2]"
                                                                     "x[3]"
```

##

sigma

```
pump$getDependencies("theta[1:3]", stochOnly = TRUE, self = FALSE)
## [1] "x[1]" "x[2]" "x[3]"
# get all dependencies, not just the direct descendants
pump$getDependencies("alpha", downstream = TRUE)
   [1] "alpha"
                     "theta[1]"
                                  "theta[2]"
                                               "theta[3]"
                                                            "theta[4]"
   [6] "theta[5]"
                     "theta[6]"
                                  "theta[7]"
                                               "theta[8]"
                                                            "theta[9]"
## [11] "theta[10]" "lambda[1]" "lambda[2]"
                                               "lambda[3]"
                                                            "lambda[4]"
## [16] "lambda[5]" "lambda[6]" "lambda[7]"
                                               "lambda[8]"
                                                            "lambda[9]"
                                                            "x[4]"
## [21] "lambda[10]" "x[1]"
                                  "x[2]"
                                               "x[3]"
## [26] "x[5]"
                     "x[6]"
                                  "x[7]"
                                               "x[8]"
                                                            "x[9]"
## [31] "x[10]"
pump$getDependencies("alpha", downstream = TRUE, dataOnly = TRUE)
## [1] "x[1]" "x[2]" "x[3]" "x[4]" "x[5]" "x[6]" "x[7]" "x[8]" "x[9]"
## [10] "x[10]"
In addition, one can determine parent nodes using getParents.
pump$getParents("alpha")
## character(0)
```

13.2 Accessing information about nodes and variables

13.2.1 Getting distributional information about a node

We briefly demonstrate some of the functionality for information about a node here, but refer readers to the R help on modelBaseClass for full details.

Here is an example model, with use of various functions to determine information about nodes or variables.

```
## "dunif"
m$isDiscrete(c('y', 'mu', 'sigma'))
## y[1] y[2] y[3] y[4]
                              mu sigma
## FALSE FALSE FALSE FALSE FALSE
m$isDeterm('mu')
##
      mu
## FALSE
m$getDimension('mu')
## value
##
m$getDimension('mu', includeParams = TRUE)
## value
          mean
                  sd
                       tau
                             var
##
                   0
                         0
                               0
```

Note that any variables provided to these functions are expanded into their constituent node names, so the length of results may not be the same length as the input vector of node and variable names. However the order of the results should be preserved relative to the order of the inputs, once the expansion is accounted for.

13.2.2 Getting information about a distribution

One can also get generic information about a distribution based on the name of the distribution using the function getDistributionInfo. In particular, one can determine whether a distribution was provided by the user (isUserDefined), whether a distribution provides CDF and quantile functions (pqDefined), whether a distribution is a discrete distribution (isDiscrete), the parameter names (include alternative parameterizations) for a distribution (getParamNames), and the dimension of the distribution and its parameters (getDimension). For more extensive information, please see the R help for getDistributionInfo.

13.2.3 Getting distribution parameter values for a node

The function getParam provides access to values of the parameters of a node's distribution. The two arguments must be the name of one (stochastic) node and the name of a parameter for the distribution followed by that node. The parameter does not have to be one of the parameters used when the node was declared. Alternative parameterization values can also be obtained. See Section 5.2.4.1 for available parameterizations. (These can also be seen in nimble:::distributionsInputList.)

Here is an example:

```
gammaModel <- nimbleModel(
    nimbleCode({
        a ~ dlnorm(0, 1)
        x ~ dgamma(shape = 2, scale = a)
    }), data = list(x = 2.4), inits = list(a = 1.2))
gammaModel$getParam('x', 'scale')</pre>
```

```
## [1] 1.2
```

```
gammaModel$getParam('x', 'rate')
```

[1] 0.8333333

getParam is part of the NIMBLE language, so it can be used in run code of nimbleFunctions.

13.2.4 Getting distribution bounds for a node

The function getBound provides access to the lower and upper bounds of the distribution for a node. In most cases these bounds will be fixed based on the distribution, but for the uniform distribution the bounds are the parameters of the distribution, and when truncation is used (Section 5.2.7), the bounds will be determined by the truncation. Like the functions described in the previous section, getBound can be used as global function taking a model as the first argument, or it can be used as a model member function. The next two arguments must be the name of one (stochastic) node and either "lower" or "upper" indicating whether the lower or upper bound is desired. For multivariate nodes the bound is a scalar that is the bound for all elements of the node, as we do not handle truncation for multivariate nodes.

Here is an example:

```
exampleModel <- nimbleModel(</pre>
    nimbleCode({
        y ~ T(dnorm(mu, sd = sig), a, Inf)
        a \sim dunif(-1, b)
        b \sim dgamma(1, 1)
    ), inits = list(a = -0.5, mu = 1, sig = 1, b = 4),
        data = list(y = 4))
getBound(exampleModel, 'y', 'lower')
## [1] -0.5
getBound(exampleModel, 'y', 'upper')
## [1] Inf
exampleModel$b <- 3
exampleModel$calculate(exampleModel$getDependencies('b'))
## [1] -4.386294
getBound(exampleModel, 'a', 'upper')
## [1] 3
exampleModel$getBound('b','lower')
```

[1] 0

getBound is part of the NIMBLE language, so it can be used in run code of nimbleFunctions. In fact, we anticipate that most use of getBound will be for algorithms, such as for the reflection version of the random walk MCMC sampler.

13.3 Carrying out model calculations

13.3.1 Core model operations: calculation and simulation

The four basic ways to operate a model are to calculate nodes, simulate into nodes, get the log probabilities (or probability densities) that have already been calculated, and compare the log probability of a new value to that of an old value. In more detail:

- calculate For a stochastic node, calculate determines the log probability value, stores it in the appropriate logProb variable, and returns it. For a deterministic node, calculate executes the deterministic calculation and returns 0.
- simulate For a stochastic node, simulate generates a random draw. For deterministic nodes, simulate is equivalent to calculate without returning 0. simulate always returns NULL (or void in C++).
- **getLogProb getLogProb** simply returns the most recently calculated log probability value, or 0 for a deterministic node.
- calculateDiff calculateDiff is identical to calculate, but it returns the new log probability value minus the one that was previously stored. This is useful when one wants to change the value or values of node(s) in the model (e.g., by setting a value or simulate) and then determine the change in the log probability, such as needed for a Metropolis-Hastings acceptance probability.

Each of these functions is accessed as a member function of a model object, taking a vector of node names as an argument¹. If there is more than one node name, calculate and getLogProb return the sum of the log probabilities from each node, while calculateDiff returns the sum of the new values minus the old values. Next we show an example using simulate.

13.3.1.1 Example: simulating arbitrary collections of nodes

```
mc <- nimbleCode({
    a ~ dnorm(0, 0.001)
    for(i in 1:5) {
        y[i] ~ dnorm(a, 0.1)
        for(j in 1:3)
            z[i,j] ~ dnorm(y[i], sd = 0.1)
    }
    y.squared[1:5] <- y[1:5]^2
})

model <- nimbleModel(mc, data = list(z = matrix(rnorm(15), nrow = 5)))

model$a <- 1
model$y</pre>
```

[1] NA NA NA NA NA

```
model$simulate("y[1:3]")
# simulate(model, "y[1:3]")
```

¹Standard usage is model\$calculate(nodes) but calculate(model, nodes) is synonymous.

```
model$y
## [1] -4.3723097 1.7331821 0.6234022
                                                NA
                                                            NA
model$simulate("y")
model$y
        6.6051146 2.0859962 -0.9702564 -0.3898915 -0.5978887
## [1]
model$z
##
              [,1]
                         [,2]
                                      [,3]
## [1,] -1.0314207 -0.9537793 -0.79453166
## [2,] -2.3710229 -0.3980044 -0.30881797
## [3,] -0.3245763 -0.3112171
                              0.36144477
## [4,] -0.9442988 0.7960927 1.39879110
## [5,] -0.7658900 0.9864283 -0.05607042
model$simulate(c("y[1:3]", "z[1:5, 1:3]"))
model$y
## [1]
        1.5882948 1.5578821 3.8961663 -0.3898915 -0.5978887
model$z
##
              [,1]
                         [,2]
                                      [,3]
## [1,] -1.0314207 -0.9537793 -0.79453166
## [2,] -2.3710229 -0.3980044 -0.30881797
## [3,] -0.3245763 -0.3112171 0.36144477
## [4,] -0.9442988 0.7960927
                              1.39879110
## [5,] -0.7658900 0.9864283 -0.05607042
model$simulate(c("z[1:5, 1:3]"), includeData = TRUE)
model$z
##
              [,1]
                         [,2]
                                    [,3]
                   1.5516260
        1.6203124
## [1,]
                               1.4942336
## [2,]
        1.6213524
                   1.5516332
                               1.5761658
## [3,] 4.0065304 4.0713699
                               3.8007847
## [4,] -0.2254835 -0.4765649 -0.3632563
## [5,] -0.5756517 -0.6255796 -0.4584634
```

The example illustrates a number of features:

- 1. simulate(model, nodes) is equivalent to model\$simulate(nodes). You can use either, but the latter is encouraged and the former may be deprecated in the future.
- 2. Inputs like "y[1:3]" are automatically expanded into c("y[1]", "y[2]", "y[3]"). In fact, simply "y" will be expanded into all nodes within y.
- 3. An arbitrary number of nodes can be provided as a character vector.
- 4. Simulations will be done in the order provided, so in practice the nodes should often be obtained by functions such as **getDependencies**. These return nodes in topologically-sorted order, which means no node is manipulated before something it depends on.

5. The data nodes z were not simulated into until includeData = TRUE was used.

Use of calculate, calculateDiff and getLogProb are similar to simulate, except that they return a value (described above) and they have no includeData argument.

13.3.2 Pre-defined nimbleFunctions for operating on model nodes: simNodes, calcNodes, and qetLoqProbNodes

simNodes, calcNodes and getLogProbNodes are basic nimbleFunctions that simulate, calculate, or get the log probabilities (densities), respectively, of the same set of nodes each time they are called. Each of these takes a model and a character string of node names as inputs. If nodes is left blank, then all the nodes of the model are used.

For simNodes, the nodes provided will be topologically sorted to simulate in the correct order. For calcNodes and getLogProbNodes, the nodes will be sorted and dependent nodes will be included. Recall that the calculations must be up to date (from a calculate call) for getLogProbNodes to return the values you are probably looking for.

```
simpleModelCode <- nimbleCode({</pre>
  for(i in 1:4){
    x[i] \sim dnorm(0,1)
    y[i] \sim dnorm(x[i], 1) # y depends on x
    z[i] ~ dnorm(y[i], 1) # z depends on y
    # z conditionally independent of x
 }
})
simpleModel <- nimbleModel(simpleModelCode, check = FALSE)</pre>
cSimpleModel <- compileNimble(simpleModel)</pre>
# simulates all the x's and y's
rSimXY <- simNodes(simpleModel, nodes = c('x', 'y') )
# calls calculate on x and its dependents (y, but not z)
rCalcXDep <- calcNodes(simpleModel, nodes = 'x')</pre>
# calls qetLoqProb on x's and y's
rGetLogProbXDep <- getLogProbNodes(simpleModel,</pre>
                   nodes = 'x')
# compiling the functions
cSimXY <- compileNimble(rSimXY, project = simpleModel)</pre>
cCalcXDep <- compileNimble(rCalcXDep, project = simpleModel)</pre>
cGetLogProbXDep <- compileNimble(rGetLogProbXDep, project = simpleModel)</pre>
cSimpleModel$x
```

```
cSimpleModel$y
## [1] NA NA NA NA
# simulating x and y
cSimXY$run()
## NULL
cSimpleModel$x
## [1] -0.6589120  0.6605313 -0.0132557 -0.9314803
cSimpleModel$y
## [1] 0.5557771 -1.4278091 -0.5394082 -2.4728829
cCalcXDep$run()
## [1] -12.46535
# gives correct answer because logProbs
# updated by 'calculate' after simulation
cGetLogProbXDep$run()
## [1] -12.46535
cSimXY$run()
## NULL
# gives old answer because logProbs
# not updated after 'simulate'
cGetLogProbXDep$run()
## [1] -12.46535
cCalcXDep$run()
## [1] -9.277808
```

13.3.3 Accessing log probabilities via *logProb* variables

model\$logProb_y

For each variable that contains at least one stochastic node, NIMBLE generates a model variable with the prefix 'logProb_'. In general users will not need to access these logProb variables directly but rather will use getLogProb. However, knowing they exist can be useful, in part because these variables can be monitored in an MCMC.

When the stochastic node is scalar, the logProb variable will have the same size. For example:

```
## [1] NA NA NA NA
model$calculate("y")
```

[1] -11.02766

model\$logProb_y

[1] -2.087536 -2.085793 -2.489620 -2.166821 -2.197894

Creation of logProb variables for stochastic multivariate nodes is trickier, because they can represent an arbitrary block of a larger variable. In general NIMBLE records the logProb values using the lowest possible indices. For example, if x[5:10, 15:20] follows a Wishart distribution, its log probability (density) value will be stored in logProb_x[5, 15]. When possible, NIMBLE will reduce the dimensions of the corresponding logProb variable. For example, in

```
for(i in 1:10) x[i,] ~ dmnorm(mu[], prec[,])
```

x may be 10×20 (dimensions must be provided), but $logProb_x$ will be 10×1 . For the most part you do not need to worry about how NIMBLE is storing the log probability values, because you can always get them using getLogProb.

Chapter 14

Data structures in NIMBLE

NIMBLE provides several data structures useful for programming.

We'll first describe *modelValues*, which are containers designed for storing values for models. Then in Section 14.2 we'll describe *nimbleLists*, which have a similar purpose to lists in R, allowing you to store heterogeneous information in a single object.

modelValues can be created in either R or in nimbleFunction setup code. nimbleLists can be created in R code, in nimbleFunction setup code, and in nimbleFunction run code, from a nimbleList definition created in R or setup code. Once created, modelValues and nimbleLists can then be used either in R or in nimbleFunction setup or run code. If used in run code, they will be compiled along with the nimbleFunction.

14.1 The modelValues data structure

modelValues are containers designed for storing values for models. They may be used for model outputs or model inputs. A modelValues object will contain *rows* of variables. Each row contains one object of each variable, which may be multivariate. The simplest way to build a modelValues object is from a model object. This will create a modelValues object with the same variables as the model. Although they were motivated by models, one is free to set up a modelValues with any variables one wants.

As with the material in the rest of this chapter, modelValues objects will generally be used in nimbleFunctions that interact with models (see Chapter 15)¹. modelValues objects can be defined either in setup code or separately in R (and then passed as an argument to setup code). The modelValues object can then used in run code of nimbleFunctions.

14.1.1 Creating model Values objects

Here is a simple example of creating a modelValues object:

```
pumpModelValues = modelValues(pumpModel, m = 2)
pumpModel$x
```

[1] 5 1 5 14 3 19 1 1 4 22

¹One may want to read this section after an initial reading of Chapter 15.

pumpModelValues\$x

```
## [[1]]
## [1] NA NA NA NA NA NA NA NA NA NA
##
## [[2]]
## [1] NA NA NA NA NA NA NA NA NA
```

In this example, pumpModelValues has the same variables as pumpModel, and we set pumpModelValues to have m = 2 rows. As you can see, the rows are stored as elements of a list.

Alternatively, one can define a modelValues object manually by first defining a modelValues *configuration* via the modelValuesConf function, and then creating an instance from that configuration, like this:

```
## [[1]]
## [1] NA NA
##
## [[2]]
## [1] NA NA
```

The arguments to modelValuesConf are matching lists of variable names, types, and sizes. See help(modelValuesConf) for more details. Note that in R execution, the types are not enforced. But they will be the types created in C++ code during compilation, so they should be specified carefully.

The object returned by modelValues is an uncompiled modelValues object. When a nimbleFunction is compiled, any modelValues objects it uses are also compiled. A NIMBLE model always contains a modelValues object that it uses as a default location to store the values of its variables.

Here is an example where the **customMV** created above is used as the setup argument for a nimble-Function, which is then compiled. Its compiled modelValues is then accessed with \$.

```
# simple nimbleFunction that uses a modelValues object
resizeMV <- nimbleFunction(
   setup = function(mv){},
   run = function(k = integer()){
      resize(mv,k)})

rResize <- resizeMV(customMV)
cResize <- compileNimble(rResize)
cResize$run(5)</pre>
```

```
cCustomMV <- cResize$mv
# cCustomMV is a compiled modelValues object
cCustomMV[['a']]
## [[1]]
## [1] NA NA
## [[2]]
## [1] NA NA
##
## [[3]]
## [1] 0 0
##
## [[4]]
## [1] 0 0
##
## [[5]]
## [1] 0 0
```

Compiled modelValues objects can be accessed and altered in all the same ways as uncompiled ones. However, only uncompiled modelValues can be used as arguments to setup code in nimbleFunctions.

In the example above a modelValues object is passed to setup code, but a modelValues configuration can also be passed, with creation of modelValues object(s) from the configuration done in setup code.

14.1.2 Accessing contents of modelValues

[1] 2 4

The values in a model Values object can be accessed in several ways from R, and in fewer ways from NIMBLE.

```
# sets the first row of a to (0, 1). R only.
customMV[['a']][[1]] <- c(0,1)

# sets the second row of a to (2, 3)
customMV['a', 2] <- c(2,3)

# can access subsets of each row
customMV['a', 2][2] <- 4

# accesses all values of 'a'. Output is a list. R only.
customMV[['a']]

## [[1]]
## [[1]]
## [[1]]
## [[2]]</pre>
```

```
# sets the first row of b to a matrix with values 1. R only.
customMV[['b']][[1]] <- matrix(1, nrow = 2, ncol = 2)

# sets the second row of b. R only.
customMV[['b']][[2]] <- matrix(2, nrow = 2, ncol = 2)

# make sure the size of inputs is correct
# customMV['a', 1] <- 1:10 "
# problem: size of 'a' is 2, not 10!
# will cause problems when compiling nimbleFunction using customMV</pre>
```

Currently, only the syntax customMV["a", 2] works in the NIMBLE language, not customMV[["a"][[2]].

We can query and change the number of rows using getsize and resize, respectively. These work in both R and NIMBLE. Note that we don't specify the variables in this case: all variables in a modelValues object will have the same number of rows.

```
getsize(customMV)

## [1] 2

resize(customMV, 3)
getsize(customMV)

## [1] 3

customMV$a

## [[1]]
## [1] 0 1
##
## [[2]]
## [1] 2 4
##
## [[3]]
## [1] NA NA
```

Often it is useful to convert a modelValues object to a matrix for use in R. For example, we may want to convert MCMC output into a matrix, for use with the coda package for processing MCMC samples, or into a list. This can be done with the as.matrix and as.list methods for modelValues objects, respectively. as.matrix will generate column names from every scalar element of variables (e.g. "b[1, 1]", "b[2, 1]", etc.). The rows of the modelValues will be the rows of the matrix, with any matrices or arrays converted to a vector based on column-major ordering.

```
as.matrix(customMV, 'a') # convert 'a'

## a[1] a[2]

## [1,] 0 1

## [2,] 2 4

## [3,] NA NA
```

```
as.matrix(customMV)
                             # convert all variables
##
        a[1] a[2] b[1, 1] b[2, 1] b[1, 2] b[2, 2] c[1]
## [1,]
                 1
                          1
                                   1
                                            1
                                                     1
                                                         NA
                                            2
## [2,]
            2
                 4
                          2
                                   2
                                                     2
                                                         NA
## [3,]
           NA
                NA
                         NA
                                  NA
                                           NA
                                                   NA
                                                         NA
```

the same size and shape as the variable with an additional index for the iteration (e.g., MCMC iteration when used for MCMC output). By default iteration is the first index, but it can be the last if needed.

```
as.list will return a list with an element for each variable. Each element will be an array of
as.list(customMV, 'a')
                         # get only 'a'
## $a
##
         [,1] [,2]
## [1,]
            0
                  1
## [2,]
            2
                  4
## [3,]
           NA
                NA
as.list(customMV)
                             # get all variables
## $a
##
         [,1] [,2]
## [1,]
            0
## [2,]
            2
                  4
## [3,]
           NA
                NA
##
## $b
##
   , , 1
##
##
         [,1] [,2]
## [1,]
            1
                  1
            2
                  2
## [2,]
## [3,]
           NA
                NA
##
   , , 2
##
##
##
         [,1] [,2]
## [1,]
            1
                  1
                  2
            2
## [2,]
##
  [3,]
           NA
                NA
##
##
## $c
##
         [,1]
## [1,]
           NA
## [2,]
           NA
## [3,]
           NA
```

```
as.list(customMV, 'a', iterationAsLastIndex = TRUE) # put iteration in last index

## $a
## [,1] [,2] [,3]
## [1,] 0 2 NA
## [2,] 1 4 NA
```

If a variable is a scalar, using unlist in R to extract all rows as a vector can be useful.

```
customMV['c', 1] <- 1
customMV['c', 2] <- 2
customMV['c', 3] <- 3
unlist(customMV['c', ])</pre>
```

```
## [1] 1 2 3
```

Once we have a modelValues object, we can see the structure of its contents via the varNames and sizes components of the object.

```
customMV$varNames
```

```
## [1] "a" "b" "c"
customMV$sizes
```

```
## $a
## [1] 2
##
## $b
## [1] 2 2
##
## $c
## [1] 1
```

As with most NIMBLE objects, modelValues are passed by reference, not by value. That means any modifications of modelValues objects in either R functions or nimbleFunctions will persist outside of the function. This allows for more efficient computation, as stored values are immediately shared among nimbleFunctions.

```
alter_a <- function(mv){
    mv['a',1][1] <- 1
}
customMV['a', 1]</pre>
```

```
## [1] 0 1
alter_a(customMV)
customMV['a',1]
```

```
## [1] 1 1
```

However, when you retrieve a variable from a modelValues object, the result is a standard R list, which is subsequently passed by value, as usual in R.

14.1.2.1 Automating calculation and simulation using modelValues

The nimbleFunctions simNodesMV, calcNodesMV, and getLogProbsMV can be used to operate on a model based on rows in a modelValues object. For example, simNodesMV will simulate in the model multiple times and record each simulation in a row of its modelValues. calcNodesMV and getLogProbsMV iterate over the rows of a modelValues, copy the nodes into the model, and then do their job of calculating or collecting log probabilities (densities), respectively. Each of these returns a numeric vector with the summed log probabilities of the chosen nodes from each row. calcNodesMV will save the log probabilities back into the modelValues object if saveLP = TRUE, a run-time argument.

Here are some examples:

```
mv <- modelValues(simpleModel)
rSimManyXY <- simNodesMV(simpleModel, nodes = c('x', 'y'), mv = mv)
rCalcManyXDeps <- calcNodesMV(simpleModel, nodes = 'x', mv = mv)
rGetLogProbMany <- getLogProbNodesMV(simpleModel, nodes = 'x', mv = mv)

cSimManyXY <- compileNimble(rSimManyXY, project = simpleModel)
cCalcManyXDeps <- compileNimble(rCalcManyXDeps, project = simpleModel)
cGetLogProbMany <- compileNimble(rGetLogProbMany, project = simpleModel)
cSimManyXY$run(m = 5) # simulating 5 times

## NULL
cCalcManyXDeps$run(saveLP = TRUE) # calculating

## [1] -18.52723 -14.93433 -23.21395 -22.64131 -12.31624
cGetLogProbMany$run() #

## [1] -18.52723 -14.93433 -23.21395 -22.64131 -12.31624
result <- as.matrix(cSimManyXY$mv) # extract simulated values</pre>
```

14.2 The nimbleList data structure

nimbleLists provide a container for storing different types of objects in NIMBLE, similar to the list data structure in R. Before a nimbleList can be created and used, a *definition*² for that nimbleList must be created that provides the names, types, and dimensions of the elements in the nimbleList. nimbleList definitions must be created in R (either in R's global environment or in setup code), but the nimbleList instances can be created in run code.

Unlike lists in R, nimbleLists must have the names and types of all list elements provided by a definition before the list can be used. A nimbleList definition can be made by using the nimbleList function in one of two manners. The first manner is to provide the nimbleList function with a series of expressions of the form name = type(nDim), similar to the specification of run-time arguments to nimbleFunctions. The types allowed for a nimbleList are the same as those allowed as run-time

²The configuration for a modelValues object is the same concept as a definition here; in a future release of NIMBLE we may make the usage more consistent between modelValues and nimbleLists.

arguments to a nimbleFunction, described in Section 11.4. For example, the following line of code creates a nimbleList definition with two elements: x, which is a scalar integer, and Y, which is a matrix of doubles.

```
exampleNimListDef <- nimbleList(x = integer(0), Y = double(2))</pre>
```

The second method of creating a nimbleList definition is by providing an R list of nimbleType objects to the nimbleList() function. A nimbleType object can be created using the nimbleType function, which must be provided with three arguments: the name of the element being created, the type of the element being created, and the dim of the element being created. For example, the following code creates a list with two nimbleType objects and uses these objects to create a nimbleList definition.

Creating definitions using a list of nimbleTypes can be useful, as it allows for programmatic generation of nimbleList elements.

Once a nimbleList definition has been created, new instances of nimbleLists can be made from that definition using the new member function. The new function can optionally take initial values for the list elements as arguments. Below, we create a new nimbleList from our exampleNimListDef and specify values for the two elements of our list:

```
exampleNimList \leftarrow exampleNimListDef$new(x = 1, Y = diag(2))
```

Once created, nimbleList elements can be accessed using the \$ operator, just as with lists in R. For example, the value of the x element of our exampleNimbleList can be set to 7 using

```
exampleNimList$x <- 7
```

nimbleList definitions can be created either in R's global environment or in setup code of a nimble-Function. Once a nimbleList definition has been made, new instances of nimbleLists can be created using the new function in R's global environment, in setup code, or in run code of a nimbleFunction.

nimbleLists can also be passed as arguments to run code of nimbleFunctions and returned from nimbleFunctions. To use a nimbleList as a run function argument, the name of the nimbleList definition should be provided as the argument type, with a set of parentheses following. To return a nimbleList from the run code of a nimbleFunction, the returnType of that function should be the name of the nimbleList definition, again using a following set of parentheses.

Below, we demonstrate a function that takes the exampleNimList as an argument, modifies its Y element, and returns the nimbleList.

```
mynf <- nimbleFunction(
  run = function(vals = exampleNimListDef()){
   onesMatrix <- matrix(value = 1, nrow = 2, ncol = 2)
   vals$Y <- onesMatrix
  returnType(exampleNimListDef())</pre>
```

```
return(vals)
})

# pass exampleNimList as argument to mynf
mynf(exampleNimList)

## nimbleList object of type nfRefClass_58

## Field "x":
## [1] 7

## Field "Y":
## [,1] [,2]

## [1,] 1 1

## [2,] 1 1
```

nimbleList arguments to run functions are passed by reference — this means that if an element of a nimbleList argument is modified within a function, that element will remain modified when the function has finished running. To see this, we can inspect the value of the Y element of the exampleNimList object and see that it has been modified.

exampleNimList\$Y

```
## [,1] [,2]
## [1,] 1 1
## [2,] 1 1
```

In addition to storing basic data types, nimbleLists can also store other nimbleLists. To achieve this, we must create a nimbleList definition that declares the types of nested nimbleLists a nimbleList will store. Below, we create two types of nimbleLists: the first, named innerNimList, will be stored inside the second, named outerNimList:

Note that definitions for inner, or nested, nimbleLists must be created before the definition for an outer nimbleList.

14.2.1 Using *eigen* and *svd* in nimbleFunctions

NIMBLE has two linear algebra functions that return nimbleLists. The eigen function takes a symmetric matrix, x, as an argument and returns a nimbleList of type eigenNimbleList. nimbleLists of type eigenNimbleList have two elements: values, a vector with the eigenvalues of x, and vectors, a square matrix with the same dimension as x whose columns are the eigenvectors of x. The eigen

function has two additional arguments: symmetric and only.values. The symmetric argument can be used to specify if x is a symmetric matrix or not. If symmetric = FALSE (the default value), x will be checked for symmetry. Eigendecompositions in NIMBLE for symmetric matrices are both faster and more accurate. Additionally, eigendecompositions of non-symmetric matrices can have complex entries, which are not supported by NIMBLE. If a complex entry is detected, NIMBLE will issue a warning and that entry will be set to NaN. The only.values arument defaults to FALSE. If only.values = TRUE, the eigen function will not calculate the eigenvectors of x, leaving the vectors nimbleList element empty. This can reduce calculation time if only the eigenvalues of x are needed.

The svd function takes an $n \times p$ matrix x as an argument, and returns a nimbleList of type svdNimbleList. nimbleLists of type svdNimbleList have three elements: d, a vector with the singular values of x, u a matrix with the left singular vectors of x, and v, a matrix with the right singular vectors of x. The svd function has an optional argument vectors which defaults to a value of "full". The vectors argument can be used to specify the number of singular vectors that are returned. If vectors = "full", v will be an $n \times n$ matrix and u will be an $p \times p$ matrix. If vectors = "thin", v will be an $n \times n$ matrix, where $m = \min(n, p)$, and u will be an $n \times p$ matrix. If vectors = "none", the u and v elements of the returned nimbleList will not be populated.

nimbleLists created by either eigen or svd can be returned from a nimbleFunction, using returnType(eigenNimbleList()) or returnType(svdNimbleList()) respectively. nimbleLists created by eigen and svd can also be used within other nimbleLists by specifying the nimbleList element types as eigenNimbleList() and svdNimbleList(). The below example demonstrates the use of eigen and svd within a nimbleFunction.

```
eigenListFunctionGenerator <- nimbleFunction(</pre>
  setup = function(){
    demoMatrix \leftarrow diag(4) + 2
    eigenAndSvdListDef <- nimbleList(demoEigenList = eigenNimbleList(),</pre>
                                        demoSvdList = svdNimbleList())
    eigenAndSvdList <- eigenAndSvdListDef$new()</pre>
  },
 run = function(){
    # we will take the eigendecomposition and sud of a symmetric matrix
    eigenAndSvdList$demoEigenList <<- eigen(demoMatrix, symmetric = TRUE,
                                               only.values = TRUE)
    eigenAndSvdList$demoSvdList <<- svd(demoMatrix, vectors = 'none')</pre>
    returnType(eigenAndSvdListDef())
    return(eigenAndSvdList)
 })
eigenListFunction <- eigenListFunctionGenerator()</pre>
outputList <- eigenListFunction$run()</pre>
outputList$demoEigenList$values
```

```
outputList$demoSvdList$d
```

```
## [1] 9 1 1 1
```

[1] 9 1 1 1

The eigenvalues and singular values returned from the above function are the same since the matrix being decomposed was symmetric. However, note that both eigendecompositions and singular value decompositions are numerical procedures, and computed solutions may have slight differences even for a symmetric input matrix.

Chapter 15

Writing nimbleFunctions to interact with models

15.1 Overview

When you write an R function, you say what the input arguments are, you provide the code for execution, and in that code you give the value to be returned¹. Using the function keyword in R triggers the operation of creating an object that is the function.

Creating nimbleFunctions is similar, but there are two kinds of code and two steps of execution:

- 1. Setup code is provided as a regular R function, but the programmer does not control what it returns. Typically the inputs to setup code are objects like a model, a vector of nodes, a modelValues object or a modelValues configuration, or another nimbleFunction. The setup code, as its name implies, sets up information for run-time code. It is executed in R, so it can use any aspect of R.
- 2. Run code is provided in the NIMBLE language, which was introduced in Chapter 11. This is similar to a narrow subset of R, but it is important to remember that it is different defined by what can be compiled and much more limited. Run code can use the objects created by the setup code. In addition, some information on variable types must be provided for input arguments, the return value, and in some circumstances for local variables. There are two kinds of run code:
 - a. There is always a primary function, given as the argument run^2 .
 - b. There can optionally be other functions, or 'methods' in the language of object-oriented programming, that share the same objects created by the *setup* function.

Here is a small example to fix ideas:

```
logProbCalcPlus <- nimbleFunction(
    setup = function(model, node) {
        dependentNodes <- model$getDependencies(node)
        valueToAdd <- 1</pre>
```

¹Normally this is the value of the last evaluated code, or the argument to return.

²This can be omitted if you don't need it.

```
},
    run = function(P = double(0)) {
        model[[node]] <-- P + valueToAdd
        return(model$calculate(dependentNodes))
        returnType(double(0))
    })
code <- nimbleCode({</pre>
    a \sim dnorm(0, 1)
    b \sim dnorm(a, 1)
})
testModel <- nimbleModel(code, check = FALSE)</pre>
logProbCalcPlusA <- logProbCalcPlus(testModel, "a")</pre>
testModel$b <- 1.5
logProbCalcPlusA$run(0.25)
## [1] -2.650377
dnorm(1.25,0,1,TRUE)+dnorm(1.5,1.25,1,TRUE) # direct validation
## [1] -2.650377
```

```
## [1] 1.25
```

testModel\$a # "a" was set to 0.5 + valueToAdd

The call to the R function called nimbleFunction returns a function, similarly to defining a function in R. That function, logProbCalcPlus, takes arguments for its setup function, executes it, and returns an object, logProbCalcPlusA, that has a *run* member function (method) accessed by \$run. In this case, the setup function obtains the stochastic dependencies of the node using the getDependencies member function of the model (see Section 13.1.3) and stores them in dependentNodes. In this way, logProbCalcPlus can adapt to any model. It also creates a variable, valueToAdd, that can be used by the nimbleFunction.

The object logProbCalcPlusA, returned by logProbCalcPlus, is permanently bound to the results of the processed setup function. In this case, logProbCalcPlusA\$run takes a scalar input value, P, assigns P + valueToAdd to the given node in the model, and returns the sum of the log probabilities of that node and its stochastic dependencies³. We say logProbCalcPlusA is an 'instance' of logProbCalcPlus that is 'specialized' or 'bound' to a and testModel. Usually, the setup code will be where information about the model structure is determined, and then the run code can use that information without repeatedly, redundantly recomputing it. A nimbleFunction can be called repeatedly (one can think of it as a generator), each time returning a specialized nimbleFunction.

Readers familiar with object-oriented programming may find it useful to think in terms of class definitions and objects. nimbleFunction creates a class definition. Each specialized nimbleFunction is one object in the class. The setup arguments are used to define member data in the object.

³Note the use of the global assignment operator to assign into the model. This is necessary for assigning into variables from the **setup** function, at least if you want to avoid warnings from R. These warnings come from R's reference class system.

15.2 Using and compiling nimbleFunctions

To compile the nimbleFunction, together with its model, we use compileNimble:

```
CnfDemo <- compileNimble(testModel, logProbCalcPlusA)
CtestModel <- CnfDemo$testModel
ClogProbCalcPlusA <- CnfDemo$logProbCalcPlusA</pre>
```

These have been initialized with the values from their uncompiled versions and can be used in the same way:

```
CtestModel$a
                   # values were initialized from testModel
## [1] 1.25
CtestModel$b
## [1] 1.5
lpA <- ClogProbCalcPlusA$run(1.5)</pre>
lpA
## [1] -5.462877
# verify the answer:
dnorm(CtestModel$b, CtestModel$a, 1, log = TRUE) +
    dnorm(CtestModel$a, 0, 1, log = TRUE)
## [1] -5.462877
CtestModel$a
                    # a was modified in the compiled model
## [1] 2.5
testModel$a
                    # the uncompiled model was not modified
## [1] 1.25
```

15.3 Writing setup code

15.3.1 Useful tools for setup functions

The setup function is typically used to determine information on nodes in a model, set up model-Values or nimbleList objects, set up (nested) nimbleFunctions or nimbleFunctionLists, and set up any persistent numeric objects. For example, the setup code of an MCMC nimbleFunction creates the nimbleFunctionList of sampler nimbleFunctions. The values of numeric objects created in setup code can be modified by run code and will persist across calls.

Some of the useful tools and objects to create in setup functions include:

- vectors of node names, often from a model Often these are obtained from the getNodeNames, getDependencies, and other methods of a model, described in Sections 13.1-13.2.
- modelValues objects These are discussed in Sections 14.1 and 15.4.4.

- **nimbleList objects** New instances of **nimbleLists** can be created from a nimbleList definition in either setup or run code. See Section 14.2 for more information.
- specializations of other nimbleFunctions A useful NIMBLE programming technique is to have one nimbleFunction contain other nimbleFunctions, which it can use in its run-time code (Section 15.4.7).
- lists of other nimbleFunctions In addition to containing single other nimbleFunctions, a nimbleFunction can contain a list of other nimbleFunctions (Section 15.4.8).

If one wants a nimbleFunction that does get specialized but has empty setup code, use setup = function() {} or setup = TRUE.

15.3.2 Accessing and modifying numeric values from setup

While models and nodes created during setup cannot be modified⁴, numeric values and modelValues can be, as illustrated by extending the example from above.

```
logProbCalcPlusA$valueToAdd # in the uncompiled version

## [1] 1
logProbCalcPlusA$valueToAdd <- 2
ClogProbCalcPlusA$valueToAdd # or in the compiled version

## [1] 1
ClogProbCalcPlusA$valueToAdd <- 3
ClogProbCalcPlusA$valueToAdd <- 3
ClogProbCalcPlusA$run(1.5)

## [1] -16.46288
CtestModel$a # a == 1.5 + 3</pre>
```

[1] 4.5

15.3.3 Determining numeric types in nimbleFunctions

For numeric variables from the setup function that appear in the run function or other member functions (or are declared in setupOutputs), the type is determined from the values created by the setup code. The types created by setup code must be consistent across all specializations of the nimbleFunction. For example if X is created as a matrix (two-dimensional double) in one specialization but as a vector (one-dimensional double) in another, there will be a problem during compilation. The sizes may differ in each specialization.

Treatment of vectors of length one presents special challenges because they could be treated as scalars or vectors. Currently they are treated as scalars. If you want a vector, ensure that the length is greater than one in the setup code and then use setSize in the run-time code.

15.3.4 Control of setup outputs

Sometimes setup code may create variables that are not used in run code. By default, NIMBLE inspects run code and omits variables from setup that do not appear in run code from compila-

⁴Actually, they can be, but only for uncompiled nimbleFunctions.

tion. However, sometimes a programmer may want to force a numeric or character variable to be included in compilation, even if it is not used directly in run code. As shown below, such variables can be directly accessed in one nimbleFunction from another, which provides a way of using nimbleFunctions as general data structures. To force NIMBLE to include variables during compilation, for example X and Y, simply include

```
setupOutputs(X, Y)
```

anywhere in the setup code.

15.4 Writing run code

In Chapter 11 we described the functionality of the NIMBLE language that could be used in run code without setup code (typically in cases where no models or modelValues are needed). Next we explain the additional features that allow use of models and modelValues in the run code.

15.4.1 Driving models: calculate, calculateDiff, simulate, getLogProb

These four functions are the primary ways to operate a model. Their syntax was explained in Section 13.3. Except for getLogProb, it is usually important for the nodes vector to be sorted in topological order. Model member functions such as getDependencies and expandNodeNames will always return topologically sorted node names.

Most R-like indexing of a node vector is allowed within the argument to calculate, calculateDiff, simulate, and getLogProb. For example, all of the following are allowed:

```
myModel$calculate(nodes)
myModel$calculate(nodes[i])
myModel$calculate(nodes[1:3])
myModel$calculate(nodes[c(1,3)])
myModel$calculate(nodes[2:i])
myModel$calculate(nodes[ values(model, nodes) + 0.1 < x ])</pre>
```

Note that in the last line of code, one must have that the length of nodes is equal to that of values(model, nodes), which means that all the nodes in nodes must be scalar nodes.

Also note that one cannot create new vectors of nodes in run code. They can only be indexed within a call to calculate, calculateDiff, simulate or getLogProb.

15.4.2 Getting and setting variable and node values

15.4.2.1 Using indexing with nodes

Here is an example that illustrates getting and setting of nodes, subsets of nodes, or variables. Note the following:

- In model[[v]], v can only be a single node or variable name, not a vector of multiple nodes nor an element of such a vector (model[[nodes[i]]] does not work). The node itself may be a vector, matrix or array node.
- In fact, v can be a node-name-like character string, even if it is not actually a node in the model. See example 4 in the code below.

- One can also use model\$varName, with the caveat that varName must be a variable name. This usage would only make sense for a nimbleFunction written for models known to have a specific variable. (Note that if a is a scalar node in model, then model[['a']] will be a scalar but model\$a will be a vector of length -)
- one should use the <<- global assignment operator to assign into model nodes.

Note that NIMBLE does not allow variables to change dimensions. Model nodes are the same, and indeed are more restricted because they can't change sizes. In addition, NIMBLE distinguishes between scalars and vectors of length 1. These rules, and ways to handle them correctly, are illustrated in the following code as well as in Section 11.3.

```
code <- nimbleCode({</pre>
    z \sim dnorm(0, sd = sigma)
    sigma ~ dunif(0, 10)
    y[1:n] \sim dmnorm(zeroes[1:n], cov = C[1:5, 1:5])
})
m <- nimbleModel(code, constants = list(n = n, zeroes = rep(0, n),
                                          C = diag(n))
cm <- compileNimble(m)</pre>
nfGen <- nimbleFunction(
    setup = function(model) {
    # node1 and node2 would typically be setup arguments, so they could
    # have different values for different models. We are assigning values
    # here so the example is clearer.
        node1 <- 'sigma'
                                    # a scalar node
        node2 <- 'y[1:5]'
                                    # a vector node
        notReallyANode <- 'y[2:4]' # y[2:4] allowed even though not a node!
    },
    run = function(vals = double(1)) {
        tmp0 <- model[[node1]]</pre>
                                           # 1. tmp0 will be a scalar
        tmp1 <- model[[node2]]</pre>
                                          # 2. tmp1 will be a vector
        tmp1 <- model[[node2]] # 2. tmp1 with oe a vector
tmp2 <- model[[node2]][1] # 3. tmp2 will be a scalar
        tmp3 <- model[[notReallyANode]] # 4. tmp3 will be a vector</pre>
        tmp4 <- model$y[3:4]  # 5. hard-coded access to a model variable
        # 6. node1 is scalar so can be assigned a scalar:
        model[[node1]] <<- runif(1)</pre>
        model[[node2]][1] <<- runif(1)
                  # 7. an element of node2 can be assigned a scalar
        model[[node2]] <<- runif(length(model[[node2]]))</pre>
                  # 8. a vector can be assigned to the vector node2
        model[[node2]][1:3] <<- vals[1:3]
              # elements of node2 can be indexed as needed
        returnType(double(1))
        out <- model[[node2]] # we can return a vector</pre>
        return(out)
```

```
Rnf <- nfGen(m)
Cnf <- compileNimble(Rnf)
Cnf$run(rnorm(10))</pre>
```

```
## [1] 0.474092599 1.193691666 -0.116395472 0.627634118 0.005961335
```

Use of [[]] allows one to programmatically access a node based on a character variable containing the node name; this character variable would generally be set in setup code. In contrast, use of \$ hard codes the variable name and would not generally be suitable for nimbleFunctions intended for use with arbitrary models.

15.4.2.2 Getting and setting more than one model node or variable at a time using values

Sometimes it is useful to set a collection of nodes or variables at one time. For example, one might want a nimbleFunction that will serve as the objective function for an optimizer. The input to the nimbleFunction would be a vector, which should be used to fill a collection of nodes in the model before calculating their log probabilities. This can be done using values:

```
# get values from a set of model nodes into a vector
P <- values(model, nodes)
# or put values from a vector into a set of model nodes
values(model, nodes) <- P</pre>
```

where the first line would assign the collection of values from nodes into P, and the second would do the inverse. In both cases, values from nodes with two or more dimensions are flattened into a vector in column-wise order.

values (model, nodes) may be used as a vector in other expressions, e.g.,

```
Y <- A %*% values(model, nodes) + b
```

One can also index elements of nodes in the argument to values, in the same manner as discussed for calculate and related functions in Section 15.4.1.

Note again the potential for confusion between scalars and vectors of length 1. values returns a vector and expects a vector when used on the left-hand side of an assignment. If only a single value is being assigned, it must be a vector of length 1, not a scalar. This can be achieved by wrapping a scalar in c() when necessary. For example:

```
# c(rnorm(1)) creates vector of length one:
values(model, nodes[1]) <- c(rnorm(1))
# won't compile because rnorm(1) is a scalar
# values(model, nodes[1]) <- rnorm(1)

out <- values(model, nodes[1]) # out is a vector
out2 <- values(model, nodes[1])[1] # out2 is a scalar</pre>
```

15.4.3 Getting parameter values and node bounds

Sections 13.2.3-13.2.4 describe how to get the parameter values for a node and the range (bounds) of possible values for the node using getParam and getBound. Both of these can be used in run code.

15.4.4 Using modelValues objects

The modelValues structure was introduced in Section 14.1. Inside nimbleFunctions, modelValues are designed to easily save values from a model object during the running of a nimbleFunction. A modelValues object used in run code must always exist in the setup code, either by passing it in as a setup argument or creating it in the setup code.

To illustrate this, we will create a nimbleFunction for computing importance weights for importance sampling. This function will use two modelValues objects. propModelValues will contain a set of values simulated from the importance sampling distribution and a field propLL for their log probabilities (densities). savedWeights will contain the difference in log probability (density) between the model and the propLL value provided for each set of values.

The simplest way to pass values back and forth between models and modelValues inside of a nimbleFunction is with copy, which has the synonym nimCopy. See help(nimCopy) for argument details.

Alternatively, the values may be accessed via indexing of individual rows, using the notation mv[var, i], where mv is a modelValues object, var is a variable name (not a node name), and i is a row number. Likewise, the getsize and resize functions can be used as discussed in Section 14.1. However the function as matrix does not work in run code.

Here is a run function to use these modelValues:

```
run = function(){
    # gets the number of rows of propSamples
    m <- getsize(propModelValues)

# resized savedWeights to have the proper rows
resize(savedWeights, m)
for(i in 1:m){
    # Copying from propSamples to model.
    # Node names of propSamples and model must match!</pre>
```

Once the nimbleFunction is built, the modelValues object can be accessed using \$, which is shown in more detail below. In fact, since modelValues, like most NIMBLE objects, are reference class objects, one can get a reference to them before the function is executed and then use that reference afterwards.

```
# simple model and modelValues for example use with code above
targetModelCode <- nimbleCode({</pre>
    x \sim dnorm(0,1)
    for(i in 1:4)
        y[i] \sim dnorm(0,1)
})
# code for proposal model
propModelCode <- nimbleCode({</pre>
    x \sim dnorm(0,2)
    for(i in 1:4)
        y[i] \sim dnorm(0,2)
})
# creating the models
targetModel = nimbleModel(targetModelCode, check = FALSE)
propModel = nimbleModel(propModelCode, check = FALSE)
cTargetModel = compileNimble(targetModel)
cPropModel = compileNimble(propModel)
sampleMVConf = modelValuesConf(vars = c("x", "y", "propLL"),
    types = c("double", "double", "double"),
    sizes = list(x = 1, y = 4, propLL = 1) )
sampleMV <- modelValues(sampleMVConf)</pre>
    nimbleFunction for generating proposal sample
PropSamp_Gen <- nimbleFunction(</pre>
    setup = function(mv, propModel){
        nodeNames <- propModel$getNodeNames()</pre>
```

```
},
    run = function(m = integer() ){
        resize(mv, m)
        for(i in 1:m){
            propModel$simulate()
            nimCopy(from = propModel, to = mv, nodes = nodeNames, row = i)
            mv["propLL", i][1] <<- propModel$calculate()</pre>
        }
    }
    )
# nimbleFunction for calculating importance weights
# uses setup and run functions as defined in previous code chunk
impWeights_Gen <- nimbleFunction(setup = setup,</pre>
                                  run = run)
# making instances of nimbleFunctions
# note that both functions share the same modelValues object
RPropSamp <- PropSamp_Gen(sampleMV, propModel)</pre>
RImpWeights <- impWeights_Gen(sampleMV, targetModel, swConf)</pre>
# compiling
CPropSamp <- compileNimble(RPropSamp, project = propModel)</pre>
CImpWeights <- compileNimble(RImpWeights, project = targetModel)</pre>
# generating and saving proposal sample of size 10
CPropSamp$run(10)
## NULL
# calculating the importance weights and saving to mv
CImpWeights$run()
## NULL
# retrieving the modelValues objects
# extracted objects are C-based modelValues objects
savedPropSamp_1 = CImpWeights$propModelValues
savedPropSamp_2 = CPropSamp$mv
# Subtle note: savedPropSamp_1 and savedPropSamp_2
# both provide interface to the same compiled modelValues objects!
# This is because they were both built from sampleMV.
savedPropSamp_1["x",1]
## [1] -0.3998956
```

```
savedPropSamp_2["x",1]
## [1] -0.3998956
savedPropSamp_1["x",1] <- 0 # example of directly setting a value</pre>
savedPropSamp_2["x",1]
## [1] 0
# viewing the saved importance weights
savedWeights <- CImpWeights$savedWeights</pre>
unlist(savedWeights[["w"]])
    [1] 0.2673310 1.3081190 0.9378951 0.5024644 1.7765357 3.5155732 0.3893390
##
    [8] 0.5530012 0.5085069 2.3890876
# viewing first 3 rows -- note that savedPropSsamp_1["x", 1] was altered
as.matrix(savedPropSamp_1)[1:3, ]
##
        propLL[1]
                       x[1]
                                   y[1]
                                               y[2]
                                                           y [3]
                                                                       y [4]
## [1,] -3.689026
                   0.000000 0.4941960 0.62275962 -0.1610471
                                                                0.09637918
                   1.303486 -0.9937620 -0.03470697 0.5615520 -0.99986562
## [2,] -6.864741
## [3,] -6.199326 -1.265670 0.9577035 -0.52875091 -0.7335897 -0.02557756
```

Importance sampling could also be written using simple vectors for the weights, but we illustrated putting them in a model Values object along with model variables.

15.4.5 Using model variables and model Values in expressions

Each way of accessing a variable, node, or modelValues can be used amidst mathematical expressions, including with indexing, or passed to another nimbleFunction as an argument. For example, the following two statements would be valid:

```
model[["x[2:8, ]"]][2:4, 1:3] %*% Z

if Z is a vector or matrix, and

C[6:10] <- mv[v, i][1:5, k] + B
```

if B is a vector or matrix.

The NIMBLE language allows scalars, but models defined from BUGS code are never created as purely scalar nodes. Instead, a single node such as defined by $z \sim dnorm(0, 1)$ is implemented as a vector of length 1, similar to R. When using z via model\$z or model[["z"]], NIMBLE will try to do the right thing by treating this as a scalar. In the event of problems⁵, a more explicit way to access z is model\$z[1] or model[["z"]][1].

15.4.6 Including other methods in a nimbleFunction

Other methods can be included with the methods argument to nimbleFunction. These methods can use the objects created in setup code in just the same ways as the run function. In fact, the run function is just a default main method name. Any method can then call another method.

⁵Please tell us!

```
methodsDemo <- nimbleFunction(</pre>
    setup = function() {sharedValue <- 1},</pre>
    run = function(x = double(1)) {
        print("sharedValues = ", sharedValue, "\n")
        increment()
        print("sharedValues = ", sharedValue, "\n")
        A \leftarrow times(5)
        return(A * x)
        returnType(double(1))
    },
    methods = list(
        increment = function() {
            sharedValue <<- sharedValue + 1</pre>
        },
        times = function(factor = double()) {
            return(factor * sharedValue)
            returnType(double())
        }))
methodsDemo1 <- methodsDemo()</pre>
methodsDemo1$run(1:10)
## sharedValues = 1
##
## sharedValues = 2
         10 20 30 40 50 60 70 80 90 100
##
    [1]
methodsDemo1$sharedValue <- 1
CmethodsDemo1 <- compileNimble(methodsDemo1)</pre>
CmethodsDemo1$run(1:10)
## sharedValues = 1
##
## sharedValues = 2
    [1]
         10 20 30 40
                          50 60 70 80 90 100
```

15.4.7 Using other nimbleFunctions

One nimbleFunction can use another nimbleFunction that was passed to it as a setup argument or was created in the setup function. This can be an effective way to program. When a nimbleFunction needs to access a setup variable or method of another nimbleFunction, use \$.

```
usePreviousDemo <- nimbleFunction(
    setup = function(initialSharedValue) {
        myMethodsDemo <- methodsDemo()
    },
    run = function(x = double(1)) {
        myMethodsDemo$sharedValue <<- initialSharedValue</pre>
```

```
print(myMethodsDemo$sharedValue)
        A <- myMethodsDemo$run(x[1:5])
        print(A)
        B <- myMethodsDemo$times(10)</pre>
        return(B)
        returnType(double())
    })
usePreviousDemo1 <- usePreviousDemo(2)
usePreviousDemo1$run(1:10)
## 2
## sharedValues = 2
##
## sharedValues = 3
##
## 15 30 45 60 75
## [1] 30
CusePreviousDemo1 <- compileNimble(usePreviousDemo1)</pre>
CusePreviousDemo1$run(1:10)
## 2
## sharedValues = 2
## sharedValues = 3
##
## 15
## 30
## 45
## 60
## 75
## [1] 30
```

15.4.8 Virtual nimbleFunctions and nimbleFunctionLists

Often it is useful for one nimbleFunction to have a list of other nimbleFunctions, all of whose methods have the same arguments and return types. For example, NIMBLE's MCMC engine contains a list of samplers that are each nimbleFunctions.

To make such a list, NIMBLE provides a way to declare the arguments and return types of methods: virtual nimbleFunctions created by nimbleFunctionVirtual. Other nimbleFunctions can inherit from virtual nimbleFunctions, which in R is called 'containing' them. Readers familiar with object oriented programming will recognize this as a simple class inheritance system. In Version 0.13.2 it is limited to simple, single-level inheritance.

Here is how it works:

```
baseClass <- nimbleFunctionVirtual(</pre>
    run = function(x = double(1)) {returnType(double())},
    methods = list(
        foo = function() { returnType(double()) }
    ))
derived1 <- nimbleFunction(</pre>
    contains = baseClass,
    setup = function(){},
    run = function(x = double(1)) {
        print("run 1")
        return(sum(x))
        returnType(double())
    },
    methods = list(
        foo = function() {
            print("foo 1")
            return(rnorm(1, 0, 1))
            returnType(double())
        }
    )
)
derived2 <- nimbleFunction(</pre>
    contains = baseClass,
    setup = function(){},
    run = function(x = double(1)) {
        print("run 2")
        return(prod(x))
        returnType(double())
    },
    methods = list(
        foo = function() {
            print("foo 2")
            return(runif(1, 100, 200))
            returnType(double())
        }
    )
)
useThem <- nimbleFunction(</pre>
    setup = function() {
        nfl <- nimbleFunctionList(baseClass)</pre>
        nfl[[1]] <- derived1()</pre>
        nfl[[2]] <- derived2()
    },
    run = function(x = double(1)) {
```

```
for(i in seq_along(nfl)) {
             print( nfl[[i]]$run(x) )
            print( nfl[[i]]$foo() )
        }
    }
    )
useThem1 <- useThem()</pre>
set.seed(1)
useThem1$run(1:5)
## run 1
## 15
## foo 1
## -0.6264538
## run 2
## 120
## foo 2
## 157.2853
CuseThem1 <- compileNimble(useThem1)</pre>
set.seed(1)
CuseThem1$run(1:5)
## run 1
## 15
## foo 1
## -0.626454
## run 2
## 120
## foo 2
## 157.285
## NULL
```

One can also use seq_along with nimbleFunctionLists (and only with nimbleFunctionLists). As in R, seq_along(myFunList) is equivalent to 1:length(myFunList) if the length of myFunList is greater than zero. It is an empty sequence if the length is zero.

Virtual nimbleFunctions cannot define setup values to be inherited.

15.4.9 Character objects

NIMBLE provides limited uses of character objects in run code. Character vectors created in setup code will be available in run code, but the only thing you can really do with them is include them in a print or stop statement.

Note that character vectors of model node and variable names are processed during compilation. For example, in model [[node]], node may be a character object, and the NIMBLE compiler processes this differently than print("The node name was ", node). In the former, the NIMBLE compiler

sets up a C++ pointer directly to the node in the model, so that the character content of node is never needed in C++. In the latter, node is used as a C++ string and therefore is needed in C++.

15.4.10 User-defined data structures

Before the introduction of nimbleLists in Version 0.6-4, NIMBLE did not explicitly have user-defined data structures. An alternative way to create a data structure in NIMBLE is to use nimbleFunctions to achieve a similar effect. To do so, one can define setup code with whatever variables are wanted and ensure they are compiled using setupOutputs. Here is an example:

```
dataNF <- nimbleFunction(</pre>
    setup = function() {
        X <- 1
        Y \leftarrow as.numeric(c(1, 2))
        Z <- matrix(as.numeric(1:4), nrow = 2)</pre>
        setupOutputs(X, Y, Z)
    })
useDataNF <- nimbleFunction(</pre>
    setup = function(myDataNF) {},
    run = function(newX = double(), newY = double(1), newZ = double(2)) {
        myDataNF$X <<- newX
        myDataNF$Y <-- newY
        myDataNF$Z <-- newZ
    })
myDataNF <- dataNF()</pre>
myUseDataNF <- useDataNF(myDataNF)</pre>
myUseDataNF$run(as.numeric(100), as.numeric(100:110),
                 matrix(as.numeric(101:120), nrow = 2))
myDataNF$X
## [1] 100
myDataNF$Y
   [1] 100 101 102 103 104 105 106 107 108 109 110
myDataNF$Z
        [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]
##
## [1.]
         101
              103
                   105
                         107
                              109
                                   111 113
                                              115
                                                   117
                                                           119
## [2,]
         102
              104
                         108
                               110
                                         114
                                               116
                                                    118
                                                           120
                    106
                                    112
myUseDataNF$myDataNF$X
## [1] 100
nimbleOptions(buildInterfacesForCompiledNestedNimbleFunctions = TRUE)
CmyUseDataNF <- compileNimble(myUseDataNF)</pre>
CmyUseDataNF$run(-100, -(100:110), matrix(-(101:120), nrow = 2))
```

```
## NULL
```

```
CmyDataNF <- CmyUseDataNF$myDataNF
CmyDataNF$X

## [1] -100

CmyDataNF$Y

## [1] -100 -101 -102 -103 -104 -105 -106 -107 -108 -109 -110

CmyDataNF$Z

## [,1] [,2] [,3] [,4] [,5] [,6] [,7] [,8] [,9] [,10]

## [1,] -101 -103 -105 -107 -109 -111 -113 -115 -117 -119

## [2,] -102 -104 -106 -108 -110 -112 -114 -116 -118 -120
```

You'll notice that:

- After execution of the compiled function, access to the X, Y, and Z is the same as for the uncompiled case. This occurs because CmyUseDataNF is an interface to the compiled version of myUseDataNF, and it provides access to member objects and functions. In this case, one member object is myDataNF, which is an interface to the compiled version of myUseDataNF\$myDataNF, which in turn provides access to X, Y, and Z. To reduce memory use, NIMBLE defaults to not providing full interfaces to nested nimbleFunctions like myUseDataNF\$myDataNF. In this example we made it provide full interfaces by setting the buildInterfacesForCompiledNestedNimbleFunctions option via nimbleOptions to TRUE. If we had left that option FALSE (its default value), we could still get to the values of interest using valueInCompiledNimbleFunction(CmyDataNF, 'X')
- We need to take care that at the time of compilation, the X, Y and Z values contain doubles via as.numeric so that they are not compiled as integer objects.
- The myDataNF could be created in the setup code. We just provided it as a setup argument to illustrate that option.

15.5 Example: writing user-defined samplers to extend NIM-BLE's MCMC engine

One important use of nimbleFunctions is to write additional samplers that can be used in NIMBLE's MCMC engine. This allows a user to write a custom sampler for one or more nodes in a model, as well as for programmers to provide general samplers for use in addition to the library of samplers provided with NIMBLE.

The following code illustrates how a NIMBLE developer would implement and use a Metropolis-Hastings random walk sampler with fixed proposal standard deviation.

```
my_RW <- nimbleFunction(
    contains = sampler_BASE,

setup = function(model, mvSaved, target, control) {
    # proposal standard deviation</pre>
```

```
scale <- if(!is.null(control$scale)) control$scale else 1</pre>
    calcNodes <- model$getDependencies(target)</pre>
},
run = function() {
    # initial model logProb
    model_lp_initial <- getLogProb(model, calcNodes)</pre>
    # generate proposal
    proposal <- rnorm(1, model[[target]], scale)</pre>
    # store proposal into model
    model[[target]] <<- proposal</pre>
    # proposal model logProb
    model_lp_proposed <- model$calculate(calcNodes)</pre>
    # log-Metropolis-Hastings ratio
    log_MH_ratio <- model_lp_proposed - model_lp_initial</pre>
    # Metropolis-Hastings step: determine whether or
    # not to accept the newly proposed value
    u <- runif(1, 0, 1)
    if(u < exp(log_MH_ratio)) jump <- TRUE</pre>
    else
                                jump <- FALSE
    # keep the model and mvSaved objects consistent
    if(jump) copy(from = model, to = mvSaved, row = 1,
                      nodes = calcNodes, logProb = TRUE)
              copy(from = mvSaved, to = model, row = 1,
    else
                      nodes = calcNodes, logProb = TRUE)
},
methods = list( reset = function () {}
```

The name of this sampler function, for the purposes of using it in an MCMC algorithm, is my_RW. Thus, this sampler can be added to an exisiting MCMC configuration object conf using:

To be used within the MCMC engine, sampler functions definitions must adhere exactly to the following:

- The nimbleFunction must include the contains statement contains = sampler_BASE.
- The setup function must have the four arguments model, mvSaved, target, control, in that order.
- The run function must accept no arguments, and have no return value. Further, after execution it must leave the mvSaved modelValues object as an up-to-date copy of the values and logProb values in the model object.

• The nimbleFunction must have a member method called **reset**, which takes no arguments and has no return value.

The purpose of the setup function is generally two-fold. First, to extract control parameters from the control list; in the example, the proposal standard deviation scale. It is good practice to specify default values for any control parameters that are not provided in the control argument, as done in the example. Second, to generate any sets of nodes needed in the run function. In many sampling algorithms, as here, calcNodes is used to represent the target node(s) and dependencies up to the first layer of stochastic nodes, as this is precisely what is required for calculating the Metropolis-Hastings acceptance probability. These probability calculations are done using model\$calculate(calcNodes).

The purpose of the mvSaved modelValues object is to store the state of the model, including both node values and log probability values, as it exists before any changes are made by the sampler. This allows restoration of the state when a proposal is rejected, as can be seen in the example above. When a proposal is accepted, one should copy from the model into the mvSaved object. NIMBLE's MCMC engine expects that mvSaved contains an up-to-date copy of model values and logProb values at the end of the run code of a sampler.

Note that NIMBLE generally expects the user-defined sampler to be defined in the global environment. If you define it in a function (which would generally be the case if you are using it in the context of parallelization), one approach would be to assign the user-defined sampler to the global environment in your function:

assign('sampler_yourSampler', sampler_yourSampler, envir = .GlobalEnv)

15.5.1 User-defined samplers and posterior predictive nodes

As of version 0.13.0, NIMBLE's handling of posterior predictive nodes in MCMC sampling has changed in order to improve MCMC mixing. Samplers for nodes that are not posterior predictive nodes no longer condition on the values of the posterior predictive nodes. This is done by two changes:

- turning off posterior predictive nodes as dependencies in the use of getDependencies when building an MCMC, and
- moving all posterior predictive samplers to be last in the order of samplers used in an MCMC iteration.

The first change calls for careful consideration when writing new samplers. It is done by making buildMCMC set a NIMBLE system option (described below) that tells getDependencies to ignore posterior predictive nodes (defined as nodes that are themselves not data and have no data nodes in their entire downstream (descendant) dependency network) before it builds the samplers (i.e., runs the setup code of each sampler). That way, the setup code of all samplers that use getDependencies will automatically comply with the new system.

User-defined samplers that determine dependencies using getDependencies should automatically work in the new system (although it is worth checking). However if a user-defined sampler manually specifies dependencies, and if those dependencies include posterior predictive nodes, the MCMC results could be incorrect. Whether incorrect results occur could depend on other parts of the model structure connected to node(s) sampled by the user-defined sampler (i.e., the target node(s)) and/or posterior predictive nodes. Therefore, it is strongly recommended that all user-defined

samplers rely on getDependencies to determine which nodes depend on the target node(s).

There are several new NIMBLE system options available to take control of the behavior just described.

- getDependenciesIncludesPredictiveNodes determines whether posterior predictive nodes are included in results of getDependencies. This defaults to TRUE, so that outside of building samplers, the behavior of getDependencies should be identical to previous versions of NIMBLE.
- MCMCusePredictiveDependenciesInCalculations gives the value to which getDependenciesIncludesPredictiveNodes will be set while building samplers. This defaults to FALSE.
- MCMCorderPosteriorPredictiveSamplersLast determines whether posterior predictive samplers are moved to the end of the sampler list. This default to TRUE. Behavior prior to version 0.13.0 corresponds to FALSE.

Here are some examples of how to use these options:

- If you want to have MCMC samplers condition on posterior predictive nodes, do nimbleOptions(MCMCusePredictiveDependenciesInCalculations = TRUE).
- If you want to get identical MCMC samplers, including their ordering, as in versions prior to 0.13.0, do nimbleOptions(MCMCusePredictiveDependenciesInCalculations = TRUE) and nimbleOptions(MCMCorderPosteriorPredictiveSamplersLast = FALSE).
- If you want to experiment with the behavior of getDependencies when ignoring posterior predictive nodes, do nimbleOptions(getDependenciesIncludesPredictiveNodes = FALSE).

15.6 Copying nimbleFunctions (and NIMBLE models)

NIMBLE relies heavily on R's reference class system. When models, modelValues, and nimbleFunctions with setup code are created, NIMBLE generates a new, customized reference class definition for each. As a result, objects of these types are passed by reference and hence modified in place by most NIMBLE operations. This is necessary to avoid a great deal of copying and returning and having to reassign large objects, both in processing models and nimbleFunctions and in running algorithms.

One cannot generally copy NIMBLE models or nimbleFunctions (specializations or generators) in a safe fashion, because of the references to other objects embedded within NIMBLE objects. However, the model member function <code>newModel</code> will create a new copy of the model from the same model definition (Section 6.1.3). This new model can then be used with newly instantiated nimbleFunctions.

The reliable way to create new copies of nimbleFunctions is to re-run the R function called nimbleFunction and record the result in a new object. For example, say you have a nimbleFunction called foo and 1000 instances of foo are compiled as part of an algorithm related to a model called model1. If you then need to use foo in an algorithm for another model, model2, doing so may work without any problems. However, there are cases where the NIMBLE compiler will tell you during compilation that the second set of foo instances cannot be built from the previous compiled version. A solution is to re-define foo from the beginning – i.e. call nimbleFunction again – and then proceed with building and compiling the algorithm for model2.

15.7 Debugging nimbleFunctions

One of the main reasons that NIMBLE provides an R (uncompiled) version of each nimbleFunction is for debugging. One can call debug on nimbleFunction methods (in particular the main *run* method, e.g., debug(mynf\$run) and then step through the code in R using R's debugger. One can also insert browser calls into run code and then run the nimbleFunction from R.

In contrast, directly debugging a compiled nimbleFunction is difficult, although those familiar with running R through a debugger and accessing the underlying C code may be able to operate similarly with NIMBLE code. We often resort to using print statements for debugging compiled code. Expert users fluent in C++ may also try setting nimbleOptions(pauseAfterWritingFiles = TRUE) and adding debugging code into the generated C++ files.

15.8 Timing nimbleFunctions with run.time

If your nimbleFunctions are correct but slow to run, you can use benchmarking tools to look for bottlenecks and to compare different implementations. If your functions are very long-running (say 1ms or more), then standard R benchmarking tools may suffice, e.g. the microbenchmark package

If your nimbleFunctions are very fast, say under 1ms, then microbenchmark will be inaccurate due to R-to-C++ conversion overhead (that won't happen in your actual functions). To get timing information in C++, NIMBLE provides a run.time function that avoids the R-to-C++ overhead.

```
myMicrobenchmark <- compileNimble(nimbleFunction(
    rum = function(iters = integer(0)){
        time1 <- run.time({
            for (t in 1:iters) myCompiledFunVersion1(1.234)
        })
        time2 <- run.time({
            for (t in 1:iters) myCompiledFunVersion2(1.234)
        })
        return(c(time1, time2))
        returnType(double(1))
    }))
print(myMicroBenchmark(100000))</pre>
```

15.9 Clearing and unloading compiled objects

Sometimes it is useful to clear all the compiled objects from a project and unload the shared library produced by your C++ compiler. To do so, you can use nimble:::clearCompiled(obj) where obj is a compiled object such as a compiled model or nimbleFunction (e.g., a compiled MCMC algorithm). This will clear all compiled objects associated with your NIMBLE project. For example, if cModel is a compiled model, nimble:::clearCompiled(cModel) will clear both the model and all associated nimbleFunctions such as compiled MCMCs that use that model. Be careful, use of nimble:::clearCompiled can be dangerous. There is some risk that if you have copies of the

R objects that interfaced to compiled C++ objects that have been removed, and you attempt to use those R objects after clearing their compiled counterparts, you will crash R. We have tried to minimize that risk, but we can't guarantee safe behavior.

15.10 Reducing memory usage

NIMBLE can create a lot of objects in its processing, and some of them use R features such as reference classes that are heavy in memory usage. We have noticed that building large models can use lots of memory. To help alleviate this, we provide two options, which can be controlled via nimbleOptions.

As noted above, the option buildInterfacesForCompiledNestedNimbleFunctions defaults to FALSE, which means NIMBLE will not build full interfaces to compiled nimbleFunctions that ony appear within other nimbleFunctions. If you want access to all such nimbleFunctions, use the option buildInterfacesForCompiledNestedNimbleFunctions = TRUE. This will use more memory but can be useful for debugging.

The option clearNimbleFunctionsAfterCompiling is more drastic, and it is experimental, so 'buyer beware'. This will clear much of the contents of an uncompiled nimbleFunction object after it has been compiled in an effort to free some memory. We expect to be able to keep making NIMBLE more efficient – faster execution and lower memory use – in the future.

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